

MINIMUM WEIGHT DESIGN OF FUSELAGE TYPE STIFFENED
CIRCULAR CYLINDRICAL SHELLS SUBJECTED TO PURE
TORSION AND COMBINED TORSION WITH AXIAL COMPRESSION
WITH AND WITHOUT LATERAL PRESSURE

A THESIS

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TABLE OF CONTENTS

	Page
ACKNOWLEDGMENTS	ii
LIST OF TABLES	v
LIST OF ILLUSTRATIONS	vii
SUMMARY	ix
NOTATIONS	xi
GLOSSARY OF ABBREVIATIONS	xv
 Chapter	
I. INTRODUCTION	1
Review of Past Work	
Statement of the Problem	
II. MATHEMATICAL FORMULATION OF THE PROBLEM	10
General Instability Critical Load	
Panel Instability Critical Load	
Prebuckling Stresses	
Local Buckling of Stringers and Rings	
Skin Wrinkling	
Formulation of the Objective Function and Design Variables	
III. MINIMUM WEIGHT DESIGN SOLUTION PROCEDURES	33
Pure Torsion	
Torsion Combined with Axial Compression with or without Lateral Pressure	
IV. EXAMPLES, NUMERICAL RESULTS AND DISCUSSION	57
V. CONCLUSIONS AND RECOMMENDATIONS	90
Conclusions	
Recommendations	

TABLE OF CONTENTS (Continued)

APPENDICES	Page
A. ASSUMPTIONS IN THE STIFFENED CYLINDRICAL SHELL ANALYSIS	96
B. PROPERTIES OF STIFFENERS	97
C. Z-RANGE FOR GENERATING DATA	102
D. SOLUTION OF EIGENVALUES AND EIGENVECTORS WHEN $\lambda_1 = -\lambda_2$	103
E. ALTERNATIVE APPROACH FOR ESTIMATING \bar{K}_{scr} for COMBINED LOADING	106
F. SAMPLE DESIGN	108
G. COMPUTER PROGRAM	118
BIBLIOGRAPHY	143
VITA	147

LIST OF TABLES

Table	Page
1. Critical Stresses in Stringers	25
2. Final Minimum Weight Design Results for Example 1 with Various Stiffeners ($l_x, l_y \leq 25$ in., Minimum 3 Rings) .	59
3. Final Minimum Weight Design Results for Example 1 with Gauge Variation and Unbounded l_x, l_y with TS-RR, HS-RR and HS-HR	60
4. Final Minimum Weight Design Results for Example 2 with Various Stiffeners ($l_x, l_y \leq$ about 25)	65
5. Final Minimum Weight Design Results for Example 2 with Gauge Variation and Unbounded l_x, l_y with TS-RR and HS-RR	66
6. Design Results for Example 3 with RS-RR	73
7. Effect of Using Different Open-Section Types of Stringers for Example 3 with RR ($C_y = 1.0$) and $MG = .05$	74
8. Effect of Using HS for Example 3 with RR ($C_y = 1.0$) and $MG = .05$	77
9. Effect of Using Different Open-Section Type of Rings for Example 3 with most Efficient Stringer (TS: $k_x = .3, C_x = 1.079$) and $MG = .05$)	78
10. Effect of Using HR for Example 3 with Most Efficient Stringer (TS: $k_x = .3, C_x = 1.079$) and $MG = .05$	81
11. Final Minimum Weight Design Results of Example 3 with the Most Efficient Stiffeners (TS-RR: $k_x = .3, C_x = 1.079, C_y = 1.0$) and Varying MG	82
12. Design Results of Example 4 and Example 3 with the Most Efficient Stiffeners of Example 3 (TS-RR: $k_x = .3, C_x = 1.079, C_y = 1.0$) and $MG = .05$	86
B1. Properties of Open-Section Type of Stiffeners	99
B2. Properties of Hat Stiffener	101

LIST OF TABLES (Continued)

Table	Page
F1. Design Chart for Example 1 with RS-RR	109
F2. Design Chart for Example 3 with TS-RR	115

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LIST OF ILLUSTRATIONS

Figure	Page
1. Shell Geometry	11
2. Sign Convention	12
3. Design Chart for \tilde{w} with RS-RR ($z = 2,000$)	36
4. Minimum Weight Design of Example 1 with RS-RR	61
5. Minimum Weight Design of Example 1 with Various Types of Stiffeners ($MG = .05$, $l_x, l_y \leq 25$)	62
6. Effect of Gauge Variation and Relaxing Maximum l_x, l_y Limits on the Minimum Weight Design of Example 1 with TS-RR	63
7. Effect of Gauge Variation and Relaxing Maximum l_x, l_y Limits on the Minimum Weight Design of Example 1 with HS-RR	63
8. Effect of Gauge Variation and Relaxing Maximum l_x, l_y Limits on the Minimum Weight Design of Example 1 with HS-RR	64
9. Minimum Weight Design of Example 2 with RS-RR	67
10. Minimum Weight Design of Example 2 with Various Types of Stiffeners ($MG = .02$; $l_x, l_y \leq \text{about } 25$)	68
11. Effect of Gauge Variation and Relaxing Maximum l_x, l_y Limits on the Minimum Weight Design of Example 2 with HS-RR	68
12. Effect of Gauge Variation and Relaxing Maximum l_x, l_y Limits on the Minimum Weight Design of Example 2 with TS-RR	69
13. Minimum Weight Design of Example 3 with RS-RR	76
14. Effect of Stringer Shapes (Open-Section Type) on the Cylinder Weight Using RR ($C_y = 1.0$) for Example 3	76
15. Effect of Using Hat Stringer ($k_{x2} = 0$) with RR ($C_y = 1.0$) on the Cylinder Weight for Example 3	80

LIST OF ILLUSTRATIONS (Continued)

Figure		Page
16.	Effect of Ring Shapes on the Cylinder Weight Using Most Efficient Stringer (TS: $k_x = .3$, $C_x = 1.079$) for Example 3	80
17.	Effect of Using Hat Ring ($k_{y^2} = 0$) with the Most Efficient Stringer (TS: $k_x = .3$, $C_x = 1.079$) on the Cylinder Weight for Example 3	83
18.	Final Minimum Weight Design of Example 3 Using the Most Efficient Stiffeners (TS-RR: $k_x = .3$, $C_x = 1.079$, $C_y = 1.0$) and Varying MG	83
19.	Minimum Weight Design of Examples 4 and 3 Using the Most Efficient Stiffeners of Example 3 (TS-RR: $k_x = .3$, $C_x = 1.079$, $C_y = 1.0$) and MG = .05	88
B1.	Stiffener Geometry	98
E1.	Interaction Curve for Combined Loading	107

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SUMMARY

A procedure including a highly automated computer program for the minimum weight design of fuselage type stiffened circular cylindrical shells subjected to torsion and to torsion combined with axial compression with and without lateral pressure is developed. The statement of the problem is:

For an internally stiffened fuselage type thin circular cylindrical shell of specified material, radius and length, find the size, shape, spacings of stiffeners and the skin thickness in order that it can safely carry a prescribed pure torsion or combined torsion with axial compression with and without lateral pressure with minimum weight.

The objective function is the composite weight of the cylinder. General instability is introduced as an equality constraint which results in an augmented objective function. By a thorough scrutiny the design variables of this resulting function are reduced to a minimum number of design parameters. By using the flexible polyhedron type of simplex method and the golden section or other sequential search* technique, an unconstrained minimization of the augmented objective function is performed. This leads to design charts which are then used to produce a design with minimum weight satisfying all other inequality constraints. These inequality constraints are: limitations in the stress levels in the skin, stringers and rings so that there is no yielding of the

*By comparing values of \bar{K}_s at successive numerical values of n starting with $n = 2$.

corresponding materials, and there is no buckling in the skin, the flanges and web of the stringers. In addition, panel buckling should not take place, simultaneous failure mode occurrence should be avoided, and all dimensions should be greater than a specified value for a minimum gauge design.

This method has a number of advantages. It provides the required insight to generate new designs with minimum penalty in weight, gives complete freedom, not only to avoid the simultaneous occurrence of failure modes but even to separate them at will so as not to increase the effect of imperfection sensitivity of the system and, with small additional computer time, it allows one to consider all possible shapes of stiffeners and the effect of varying minimum gauge on the weight of the composite shell.

The procedure is demonstrated through a number of selected practical design problems.

NOTATIONS

A_x, A_y	Stringer and ring cross-sectional areas, in ²
C_x, C_y	Stringer and ring shape parameters
D	Flexural stiffness of the skin, in-lb
E, E_x, E_y	Moduli of elasticity of skin, stringer and ring, psi
$(GJ)_x, (GJ)_y$	Stringer and ring contributions to the torsional stiffness, in ² -lb
I_{xc}, I_{yc}	Centroidal moments of inertia of stringer and ring, in ⁴
$\bar{K}_s, \bar{K}_{xx}, \bar{K}_{yy}$	Torsional, axial compression and pressure buckling load coefficients
$\bar{K}_{s_{cr}}$	Critical torsional buckling load coefficient for general instability
$\bar{K}_{sp_{cr}}$	Critical torsional buckling load coefficient for panel instability
L	Total length of the shell, in
$\bar{N}_{xy}, \bar{N}_x, \bar{N}_y$	Applied stress resultants, lb/in
$\bar{N}_{xy_{cr}}$	Critical uniform torsional stress resultant for general instability, lb/in
$\bar{N}_{xyp_{cr}}$	Critical uniform torsional stress resultant for panel instability, lb/in
\bar{N}_{xy}^*	Nondimensional torsional load parameter
R	Radius of the shell, in
W	Weight of the composite shell, lb

\bar{W}	Nondimensional weight parameter
\bar{W}_{\min}	Minimum nondimensional weight parameter
W^*	Composite weight function, lb
\bar{W}^*	Nondimensional composite weight function
Z	Curvature parameter, $L^2(1-\nu^2)^{\frac{1}{2}}/Rh$
b_{fx}, b_{fy}	Stringer and ring flange widths, in.
c_{fx}, c_{fy}	Stringer and ring flange to web thickness ratios
d_{wx}, d_{wy}	Stringer and ring web depths, in
e_x, e_y	Stringer and ring eccentricities, in
\bar{e}_x, \bar{e}_y	Nondimensional stringer and ring eccentricities
h	Skin thickness, in
k_{lx}, k_{ly}	Stringer and ring stress-spacing factors
k_x, k_y	Stringer and ring width to depth ratios
l_x, l_y	Stringer and ring spacings, in
n	Number of circumferential waves in general instability
n_p	Number of circumferential waves in panel instability
p_x, p_y	Stringer and ring shape multipliers for their radii of gyration
q	Applied pressure (positive outward), psi
q_x, q_y	Stringer and ring shape multipliers for their torsional rigidities
t_{fx}, t_{fy}	Stringer and ring flange thickness, in
t_{wx}, t_{wy}	Stringer and ring web thickness, in
u, v, w	Displacement components of points on the reference surface, in
x, y, z	Coordinate directions

$\bar{\alpha}_x, \bar{\alpha}_y$	Nondimensional stringer and ring radii of gyration
β	$nL/\pi R$
ξ	Ratio of pressure to axial load coefficients, $\bar{K}_{yy}/\bar{K}_{xx}$
λ	Lagrange multiplier
λ^*	Nondimensional Lagrange multiplier
$\bar{\lambda}_{xx}, \bar{\lambda}_{yy}$	Nondimensional stringer and ring extensional stiffness parameters
$\rho_{sk}, \rho_x, \rho_y$	Weight densities of the skin, stringer and ring, lb/in ³
$\bar{\rho}_{xx}, \bar{\rho}_{yy}$	Nondimensional stringer and ring flexural stiffness parameters
ν	Poisson's ratio
σ_o	Yield stress in tension/compression, psi
$\sigma_{xx_{sk}}, \sigma_{yy_{sk}}$	x and y-directional prebuckling stresses in the skin, psi
$\sigma_{xy_{sk}}$	Prebuckling shear stress in the skin, psi
$\sigma_{xy_{sky}}$	Yield shear stress in the skin material, psi
$\sigma_{xx_{st}}, \sigma_{yy_r}$	Prebuckling stresses in stringer and ring, psi
$\sigma_{xx_{sk_{cr}}}$	Local critical stress for skin buckling in compression, psi
$\sigma_{xy_{sk_{cr}}}$	Local critical stress for skin buckling in shear, psi
$\sigma_{xx_{st_{fcr}}}, \sigma_{xx_{stw_{cr}}}$	Local critical stresses for the stringer flange and web buckling, psi
superscript "0"	Refers to membrane state
superscript "1"	Refers to the additional quantity necessary to

bring the membrane state to the classical buckling
state

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GLOSSARY OF ABBREVIATIONS

Coefficients of Buckling and Yielding

GB	Gross buckling, $\bar{N}_{xy}/\bar{N}_{xy_{cr}}$
PB	Panel buckling, $\bar{N}_{xy}/\bar{N}_{xy_p}$
RYT	Ring yielding in tension, σ_{yy_r}/σ_o
SB	Skin buckling, $\sigma_{xy_{sk}}/\sigma_{xy_{sk_{cr}}}$
STFB	Stringer flange buckling, $\sigma_{xx_{st}}/\sigma_{xx_{stf_{cr}}}$
STWB	Stringer web buckling, $\sigma_{xx_{st}}/\sigma_{xx_{stw_{cr}}}$
STYC	Stringer yielding in compression, $\sigma_{xx_{st}}/\sigma_o$
SY	Skin yielding, $\sigma_{xy_{sk}}/\sigma_{xy_{sk_y}}$

Gauge Variation

MG	Minimum gauge adopted
WMG	Without minimum gauge

Types of Stiffening

AR	Angle ring
AS	Angle stringer
AS-RR	Angle stringer and rectangular ring
CR	Channel ring
CS	Channel stringer
CS,ZS,IS-RR	Channel, Zee, I stringer and rectangular ring
HR	Hat ring
HS	Hat stringer

HS-RR	Hat stringer and rectangular ring
IAR	Inverted angle ring
IAS	Inverted angle stringer
IAS-RR	Inverted angle stringer and rectangular ring
IR	I ring
IS	I stringer
RR	Rectangular ring
RS	Rectangular stringer
RS-RR	Rectangular stringer and rectangular ring
TR	Tee ring
TS	Tee stringer
TS-AR	Tee stringer and angle ring
TS-CR,ZR,IR	Tee stringer and C, Zee, I ring
TS-HR	Tee stringer and hat ring
TS-RR	Tee stringer and rectangular ring
TS-TR	Tee stringer and tee ring
ZR	Zee ring
ZS	Zee stringer

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CHAPTER I

INTRODUCTION

A stiffened thin circular cylindrical shell is a widely used configuration in the aircraft and aerospace industry. In order to achieve higher efficiencies, many attempts have been made to make the configuration as light as possible. With man's everlasting desire to overcome the barrier of time and space and to explore the millions of unexplored planets surrounding his own, the need for efficient lightweight structures will be evergrowing.

Although the configuration cited above is, in practice, subjected to torsion as well as axial compression and uniform lateral pressure, no work is found in the literature on the minimum weight design of such structures subjected to either torsion alone or torsion combined with axial compression, with or without lateral pressure. Many investigators have considered minimum weight design under uniform compression. Few have considered axial compression combined with uniform lateral pressure. So the general trend to date has been to consider axial compression with or without lateral pressure for the fuselage type of structures, take up either conventional type longitudinal and/or circumferential stiffeners or a truss-core type of sandwich construction, analyze the effects of one or more types of stiffener geometries, follow different optimization techniques and produce a design.

Review of Past Work

Although it is not directly related to the present work, a review of some of the work done on the minimum weight design of fuselage type of structures and the methods of approach adopted by the past investigators is included here for the sake of completeness.

Crawford and Burns [1]* presented in 1963 an optimum design analysis of truss-core sandwich type circular cylinders under axial compression. Cohen [2] investigated the optimum design of single and double truss-core sandwich cylinders in compression without presenting any numerical results. Burns and Almroth [3] analyzed only external and rectangular stiffeners for axially compressed cylinders and showed that the use of both stringers and rings is always more efficient than the use of either type alone. Gerard and Papirno [4] have considered ring stiffened cylinders and have shown the effects of using flanged and unflanged stiffeners on the cylinder efficiency. In 1966, Burns and Skogh [5] considered optimum design under combined uniform axial compression and uniform lateral pressure. By taking only axial compression and rectangular stiffeners, Burns [6] studied the effects of ring-stringer eccentricities. Gerard [7] presented in 1966 a comprehensive bibliography on optimal structural design. Kicher [8] considered combined axial compression and lateral pressure and found the effects of stiffener eccentricities. A systematic design of stiffened cylinders of different geometries subjected to single axial compression and to combined axial compression and radial pressure was produced by

*Numbers in brackets refer to Bibliography.

Morrow and Schmit [9] in 1968. They treated only rectangular stiffeners. In 1969, Stroud and Sykes [10] showed the effect of slight meridional curvature on the minimum weight design of axially compressed stiffened shells. Lakshmikantham and Gerard [11] showed the effects of ring stiffeners on the isotropic skin. In 1970, Almroth, Burns and Pittner [12] produced a design criterion by taking a load reduction factor ϕ such that $N_{cr} = \phi N_{cl}$. Pappas and Amba-Rao [13] also have taken a load reduction factor ϕ and designed the shell for uniform axial compression and lateral pressure. Block [14] came up with a minimum weight design of axially compressed ring and stringer stiffened cylinders. Based on the premise that minimum weight is achieved when all possible buckling modes (general, panel and local) occur simultaneously, Shideler, Anderson and Jackson [15] analyzed an axially compressed cylinder. Jones and Hague [16] applied different optimization techniques and extended the work of Reference [9]. In 1972, a review of optimal structural design was presented by Niordson and Pedersen [17]. An indirect and trial and error design procedure for axially compressed cylinders has been proposed by Rehfield [18]. Simites and Ungbhakorn [19] and [20] have produced minimum weight designs of axially compressed cylinders by considering different opened types of stiffeners.

Two types of approaches have been adopted so far. One group of researchers (see, for example, References [1,2,3,5,6,14,15 and 18]) have contended that the minimum weight design is achieved at the simultaneous occurrence of all possible failure modes. A second group of people, by employing modern optimization techniques (penalty

functions), disproved the simultaneous failure mode contention. Schmit, Kicher, and Morrow [21] by considering integrally stiffened waffle plates in 1963 explained that the concept of simultaneous failure modes is wrong for constrained minimization problems. They also cited examples where this concept is not true. Furthermore, it has been shown that for certain structural configurations design based on the simultaneous occurrence of failure modes gives rise to imperfection sensitivity. Van der Neut [22] was the first man to come up with a concrete step in 1969 which showed that imperfection-sensitivity does indeed arise in the case of an idealized thin-walled compression member. By analyzing the buckling of a square tube Graves-Smith [23] also arrived at the same conclusion in 1969. Koiter and Kuiken [24] confirmed Van der Neut's findings in 1971 by using Koiter's general nonlinear theory of elastic stability. A quantitative study of the validity of optimization based on the simultaneous buckling mode criterion has been made by Thompson and Lewis [25] in 1972 who, by formulating a thin-walled strut design problem, have shown the extent to which the optimum based on the simultaneity of overall and local buckling can be eroded away by the imperfection-sensitivity associated with the coupled branching behavior. These effects have been shown in diagrams and explained for different types of structural members under destabilizing loads by Thompson and Hunt [43]. It is known that cylinders are imperfection sensitive; the extent of sensitivity depending on the type of loading. Since linear theory is used for the analysis, a knockdown factor may be used to account for this effect.

Due to reasons explained above, Simitzes and his collaborators [19,20,26] have avoided the simultaneous occurrence of failure modes in their minimum weight designs. This procedure is adopted in the present work. It is worthy of mention that it is strictly an accident for all failure modes to be active at the minimum weight design for a given problem. Thus the design achieved through simultaneous occurrence of failure modes does not, in general, correspond to minimum weight. On the basis of past experience [9,13,16,19,20,26] it is evident that only a small number of failure modes is active at the minimum weight configuration.

Statement of the Problem

As is obvious from above, no work is found on the minimum weight design of stiffened thin circular cylindrical shells subjected to torsion and to torsion combined with axial compression with and without lateral pressure. In practice when an aircraft is maneuvering, yawing, pitching, etc., the fuselage is subjected to torsion, axial compression (due to bending) and some lateral pressure; so are missiles, launch vehicles and other spacecrafts.

A design based on pure torsional loading is desired for the case when torsion may be the only or the predominant load. Secondly, torsion combined with axial compression is the most realistic case and thus it is essential to obtain a minimum weight design under this loading condition. Thirdly, it is equally important to investigate the effects of uniform lateral pressure over and above torsion combined with axial compression on the minimum weight design of fuselage type of structures.

Hence, the precise statement of the problem in this research work is as follows: For a fuselage-type stiffened thin circular cylindrical shell of specified material, radius and length, find the size, shape, spacings of the stiffeners and the skin thickness in order that it can safely carry prescribed pure torsion and combined torsion with axial compression with and without lateral pressure with minimum weight.

The design objective is minimum weight. The equality constraint is general instability (considered as one of the active failure modes which implies designing against general instability). The remaining failure modes, i.e., panel buckling, skin wrinkling, local instability of stringers, yielding of the skin and stiffeners, failure mode interactions and geometric constraints for a realistic (minimum gauge) design are considered as inequality constraints. An active failure mode is one which represents the principal catastrophic mode of failure. This being so, the primary consideration of the shell design must be based on the active failure mode, general instability here. The other failure modes are taken as inequality constraints since it is possible to avoid them by adjusting some of the design variables.

In general the active mode or modes of failure, that govern minimum weight design, are not known apriori. If this is the case one must use the penalty function formulation to establish the active modes. Unfortunately the state of the art in employing this approach is not very reliable [16]. In such cases one may use this formulation not to achieve a final minimum weight design but simply to establish which of the failure modes are active. Once this has been accomplished, the present formulation may be employed. In addition, on the basis of

design experience, in many cases one of the active modes of failure governing the minimum weight design is known. For example, in aircraft fuselage design general instability governs the configuration of the mid portion of the fuselage. In such cases the present formulation is not only applicable but desirable as explained at the end of this chapter.

The nature of stresses in the skin and stiffeners, their corresponding failure modes and the extent to which any particular failure mode is active highly differ from one loading case to the other for the three loading cases, namely, pure torsion, torsion combined with axial compression, and torsion combined with axial compression and lateral pressure. Besides, the minimum weight design for the combined load cases presents another unique feature in the sense that while seeking an optimum solution one has to be careful to see that the shell does not buckle with a single load at any stage of the solution. The optimization and design procedures have to be modified accordingly to take care of these differing characteristics. All these are discussed in detail in Chapters II and III. The procedures common to all the three loading cases are discussed below.

In order to minimize the weight of the composite shell under the constraint that the applied (general instability) load should have a fixed value, penalty function technique is used. This gives an augmented objective function (also called the unconstrained penalized performance function) with the equality constraint inherently built-in. The primary function to be minimized, in our case the cylinder weight, is the pay-off or the performance function and the term with

the penalty parameter (also known as weighting factor) is called the penalty function. Thus, by adding the constraint to the performance function in some form as a penalty one obtains the augmented objective function. Different forms of penalty functions have been used in practice (one who is interested should refer to any standard book on optimization or nonlinear programming). This way the constrained minimization problem is converted into an unconstrained minimization problem. The effect of the constraint on the value of the performance function is gradually diminished and in the limit when the constraint is satisfied, the value of the performance function approaches the value of the augmented objective function. The advantage of transforming the constrained problem into an unconstrained problem is that much simpler optimization algorithms can be employed.

By a thorough scrutiny the number of independent variables of the resulting augmented function is reduced to a minimum. Then an unconstrained optimization is performed to minimize the objective function w.r.t. the independent variables and the generated data are recorded. An attempt has been made to produce a highly automated design procedure with least amount of wasted computer time so as to take care of the inequality constraints. Data produced from this program are then used in a second program to finally obtain a design satisfying all the constraints with minimum weight. This method provides the designer with the required insight to generate new designs with minimum penalty in weight in view of changing constraints and requirements such as availability of material and fabrication costs. It gives full freedom to the designer, based on generated data, not

only to avoid the simultaneous occurrence of various failure modes but even to separate them at will so as not to increase the imperfection sensitivity of the system. Finally, with small additional computer time it allows one to consider all possible shapes of stiffeners and the effect of varying minimum gauge on the weight of the composite shell.

CHAPTER II

MATHEMATICAL FORMULATION OF THE PROBLEM

General Instability Critical Load

With the assumptions given in Appendix A, the coordinate system and sign conventions shown in Figures 1 and 2, and following the steps outlined in References [27] and [28], the single higher order Donnell-Batdorf type of differential equation for the composite circular cylindrical shell is given by:

$$\begin{aligned}
 L(w^1) = & \nabla_D w^1 + \left(\frac{1}{1+\bar{\rho}_{yy}} \right) \nabla_E^{-1} \left[\frac{12Z^2}{\pi^4 (1-\nu^2)} (1+\bar{\lambda}_{xx}) \nabla_C w^1 \right. \\
 & - \left(\frac{L}{\pi R} \right)^2 \bar{K}_{yy} \nabla_P w^1 \left. \right] - \left(\frac{L}{\pi} \right)^2 \left(\frac{1}{1+\bar{\rho}_{yy}} \right) \left[\left(\frac{1}{2} \bar{K}_{yy} - \bar{K}_{xx} \right) w^1_{,xx} \right. \\
 & \left. \left. + \bar{K}_{yy} w^1_{,yy} + 2 \bar{K}_s w^1_{,xy} \right] = 0 \quad (1)
 \end{aligned}$$

where the operators ∇_D , ∇_E , ∇_C and ∇_P are given by

$$\nabla_D = \left(\frac{L}{\pi} \right)^4 \left[\frac{1+\bar{\rho}_{xx}}{1+\bar{\rho}_{yy}} \frac{\partial^4}{\partial x^4} + \frac{2\Lambda_7}{1+\bar{\rho}_{yy}} \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} \right]$$

$$\nabla_E = \left(\frac{L}{\pi} \right)^4 \left[\frac{\partial^4}{\partial x^4} + \frac{\Lambda_1}{1+\bar{\lambda}_{xx}} \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{1+\bar{\lambda}_{yy}}{1+\bar{\lambda}_{xx}} \frac{\partial^4}{\partial y^4} \right]$$

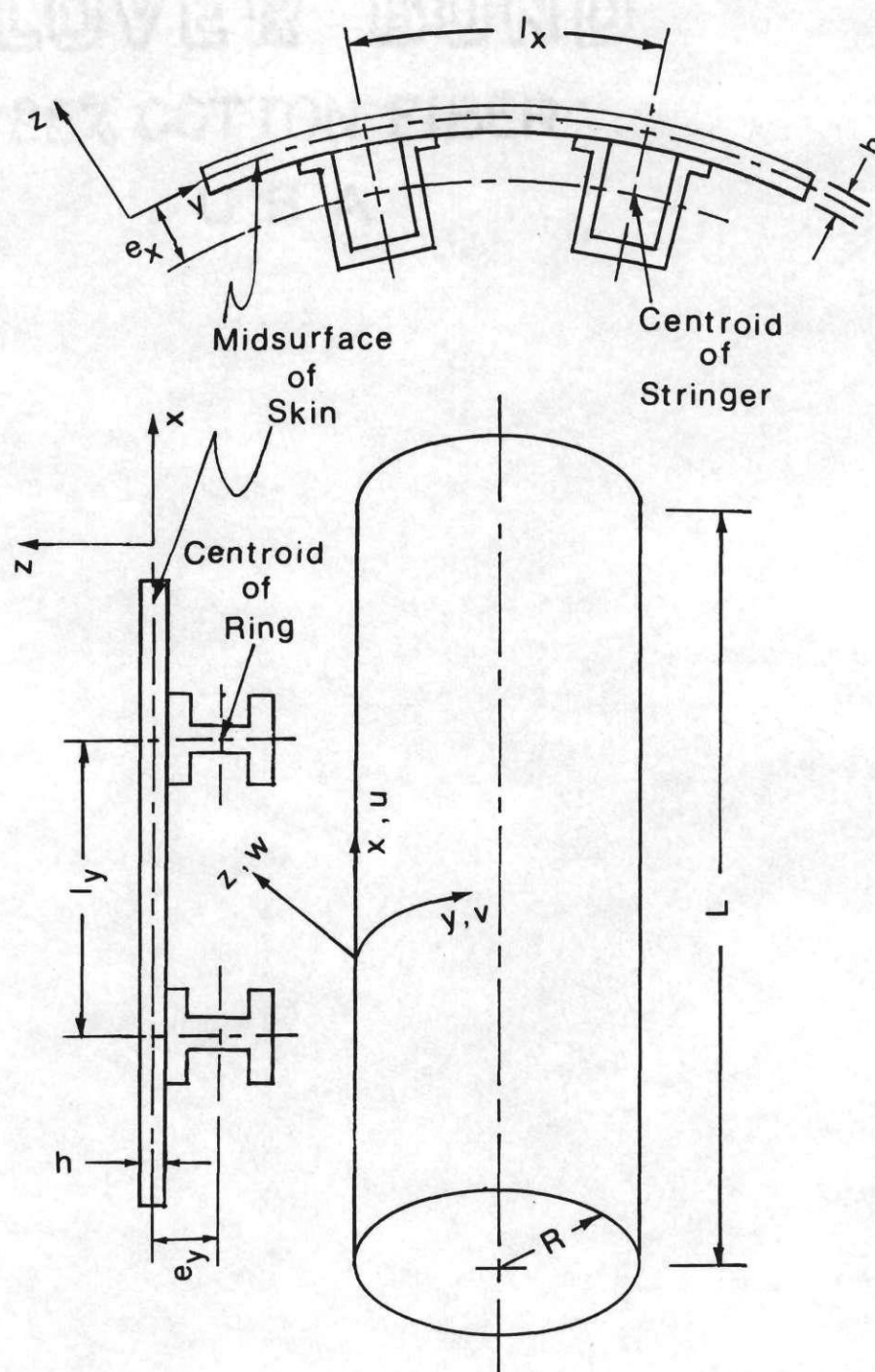


Figure 1: Shell Geometry

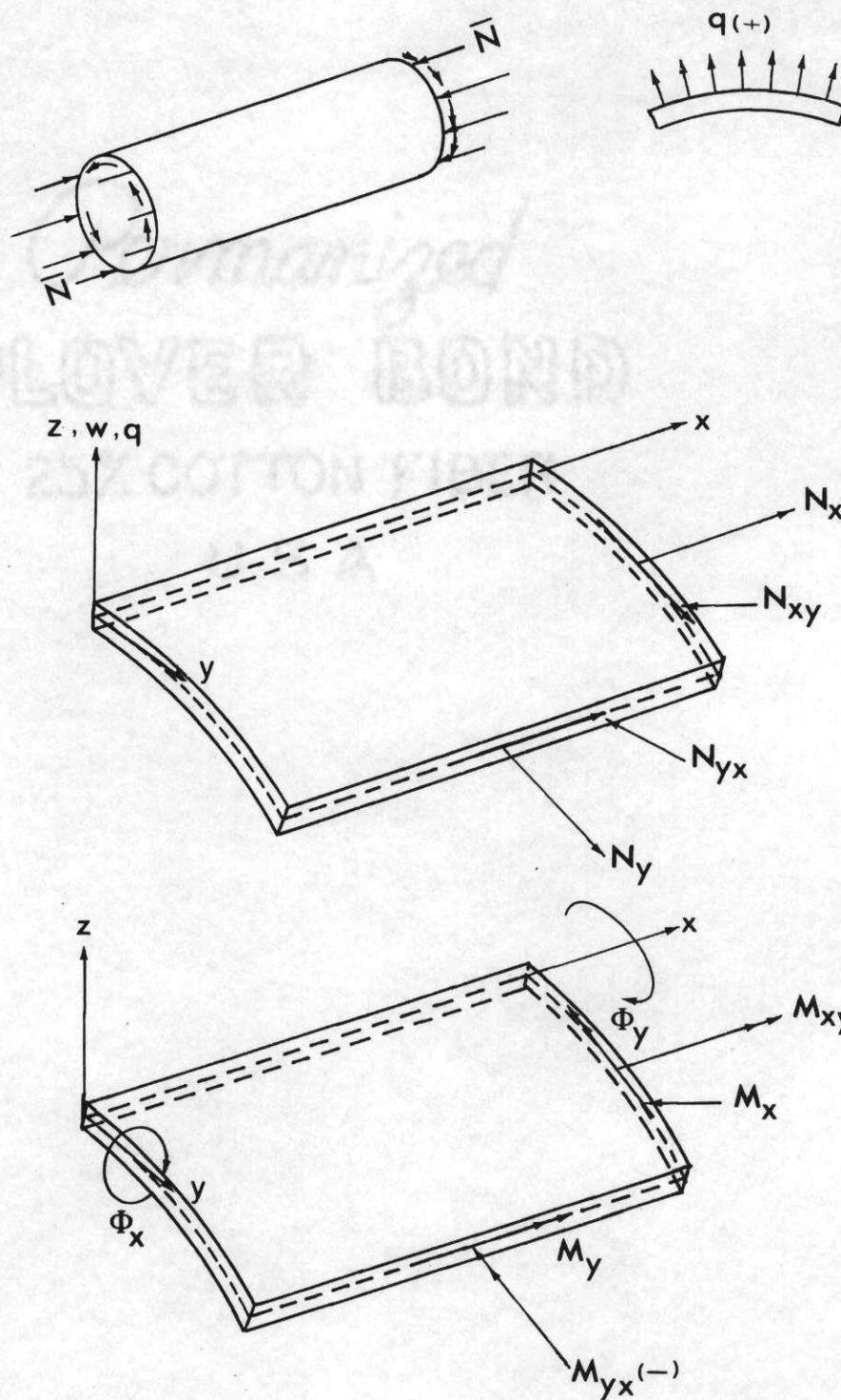


Figure 2: Sign Convention

$$\begin{aligned}
v_C = & \left(\frac{L}{\pi}\right)^8 \frac{1}{(1+\bar{\lambda}_{xx})^2} \left[\bar{\lambda}_{xx} \bar{e}_x^2 \frac{\partial^8}{\partial x^8} + \Lambda_2 \frac{\partial^8}{\partial x^6 \partial y^2} + \Lambda_3 \frac{\partial^8}{\partial x^4 \partial y^4} \right. \\
& + \Lambda_4 \frac{\partial^8}{\partial x^2 \partial y^6} + \bar{e}_y^2 \bar{\lambda}_{yy} \frac{\partial^8}{\partial y^8} + 2\nu \bar{e}_x \left(\frac{\pi}{L}\right)^2 \bar{\lambda}_{xx} \frac{\partial^6}{\partial x^6} - 2\left(\frac{\pi}{L}\right)^2 \Lambda_5 \frac{\partial^6}{\partial x^4 \partial y^2} \\
& \left. + 2\left(\frac{\pi}{L}\right)^2 \nu \bar{e}_y \bar{\lambda}_{yy} \frac{\partial^6}{\partial x^2 \partial y^4} + \left(\frac{\pi}{L}\right)^4 \Lambda_6 \frac{\partial^4}{\partial x^4} \right] \\
v_P = & \left(\frac{L}{\pi}\right)^6 \left[\bar{e}_x \frac{\bar{\lambda}_{xx}}{1+\bar{\lambda}_{xx}} \frac{\partial^6}{\partial x^6} + \Lambda_8 \frac{\partial^6}{\partial x^4 \partial y^2} + \Lambda_9 \frac{\partial^6}{\partial x^2 \partial y^4} + \bar{e}_y \frac{\bar{\lambda}_{yy}}{1+\bar{\lambda}_{xx}} \frac{\partial^6}{\partial y^6} \right. \\
& \left. - \left(\frac{\pi}{L}\right)^2 \frac{\nu}{1+\bar{\lambda}_{xx}} \frac{\partial^4}{\partial x^4} + \left(\frac{\pi}{L}\right)^2 \Lambda_{10} \frac{\partial^4}{\partial x^2 \partial y^2} - \left(\frac{\pi}{L}\right)^2 \left(\frac{1+\bar{\lambda}_{yy}}{1+\bar{\lambda}_{xx}}\right) \frac{\partial^4}{\partial y^4} \right] \quad (2)
\end{aligned}$$

and the classical simply supported boundary conditions (at $x = 0, L$) are:

$$\begin{aligned}
N_{xx}^1 &= (1+\bar{\lambda}_{xx})u_{,x}^1 + \nu(v_{,y}^1 + \frac{w^1}{R}) - \frac{L^2 \bar{e}_x}{\pi^2 R} \bar{\lambda}_{xx} w_{,xx}^1 = 0 \\
v^1 &= 0 \\
w^1 &= 0
\end{aligned} \quad (3)$$

$$M_{xx} = \left[1 + \bar{\rho}_{xx} + \frac{12Z^3 \bar{\lambda}_{xx} \bar{e}_x^2 R}{\pi^4 L^2 (1-\nu^2)^{1/2}} \right] w_{,xx}^1 - \nu w_{,yy}^1 + \frac{12Z^3 \bar{\lambda}_{xx} \bar{e}_x R^2}{\pi^2 L^4 (1-\nu^2)^{1/2}} u_{,x}^1 = 0$$

The superscript 1 parameters correspond to the additional small changes in the buckled equilibrium state over the membrane state. The constants

Λ_i ($i=1$ to 10) introduced here for brevity are given by

$$\begin{aligned}
 \Lambda_1 &= 2 \left(1 + \frac{\bar{\lambda}_{xx} + \bar{\lambda}_{yy} + \bar{\lambda}_{xx}\bar{\lambda}_{yy}}{1 - \nu} \right) \\
 \Lambda_2 &= 2\bar{e}_x^{-2}\bar{\lambda}_{xx} \left(1 + \frac{\bar{\lambda}_{yy}}{1 - \nu} \right) \\
 \Lambda_3 &= \bar{e}_x^{-2}\bar{\lambda}_{xx} + \bar{e}_y^{-2}\bar{\lambda}_{yy} + \bar{\lambda}_{xx}\bar{\lambda}_{yy} \left[\bar{e}_x^{-2} + \bar{e}_y^{-2} + 2\bar{e}_x\bar{e}_y \left(\frac{1+\nu}{1-\nu} \right) \right] \\
 \Lambda_4 &= 2\bar{e}_y^{-2}\bar{\lambda}_{yy} \left(1 + \frac{\bar{\lambda}_{xx}}{1 - \nu} \right) \\
 \Lambda_5 &= \bar{e}_x\bar{\lambda}_{xx}(1 + \bar{\lambda}_{yy}) + \bar{e}_y\bar{\lambda}_{yy}(1 + \bar{\lambda}_{xx}) \\
 \Lambda_6 &= (1 + \bar{\lambda}_{xx})(1 + \bar{\lambda}_{yy}) - \nu^2 \\
 \Lambda_7 &= 1 + \frac{(GJ)_x}{2D\ell_x} + \frac{(GJ)_y}{2D\ell_y} \\
 \Lambda_8 &= \bar{e}_x \frac{\bar{\lambda}_{xx}}{1 + \bar{\lambda}_{xx}} \left(1 - \nu + \frac{2\bar{\lambda}_{yy}}{1 - \nu} \right) \\
 \Lambda_9 &= \bar{e}_y \frac{\bar{\lambda}_{yy}}{1 + \bar{\lambda}_{xx}} \left(1 + \frac{2\bar{\lambda}_{xx}}{1 - \nu} \right) \\
 \Lambda_{10} &= \frac{(1 + \nu)(1 - \nu + \bar{\lambda}_{yy}) + 2\bar{\lambda}_{xx}(1 + \bar{\lambda}_{yy})}{(1 - \nu)(1 + \bar{\lambda}_{xx})}
 \end{aligned} \tag{4}$$

It should be noted that the following parameters have been introduced in the above equations.

$$\begin{aligned}
\bar{\lambda}_{xx} &= \frac{E_x A_x (1-\nu^2)}{Eh\ell_x} ; & \bar{\lambda}_{yy} &= \frac{E_y A_y (1-\nu^2)}{Eh\ell_y} \\
\bar{e}_x &= \left(\frac{\pi R}{L}\right)^2 \frac{e_x}{R} ; & \bar{e}_y &= \left(\frac{\pi R}{L}\right)^2 \frac{e_y}{R} \\
\bar{\rho}_{xx} &= \frac{E_x I_{xc}}{D\ell_x} ; & \bar{\rho}_{yy} &= \frac{E_y I_{yc}}{D\ell_y} \\
Z &= \frac{L^2(1-\nu^2)^{1/2}}{Rh} ; & \bar{K}_s &= \frac{\bar{N}_{xy} L^2}{\pi^2 D} \\
\bar{K}_{xx} &= \frac{\bar{N}_L^2}{\pi^2 D} ; & \bar{K}_{yy} &= \frac{qRL^2}{\pi^2 D}
\end{aligned} \tag{5}$$

and $D = Eh^3/12(1-\nu^2)$. The subscripts x and y refer to the stringers and rings respectively.

Using the definitions of various operators from Equation (2) into Equation (1), the resulting single higher order buckling equation is

$$\begin{aligned}
L(w^1) &= \left(\frac{L}{\pi}\right)^4 \left[\frac{1+\bar{\rho}_{xx}}{1+\bar{\rho}_{yy}} \frac{\partial^4 w^1}{\partial x^4} + \frac{2\Lambda_7}{1+\bar{\rho}_{yy}} \frac{\partial^4 w^1}{\partial x^2 \partial y^2} + \frac{\partial^4 w^1}{\partial y^4} \right] \\
&+ \left(\frac{L}{\pi}\right)^4 \frac{12Z^2}{\pi^4(1-\nu^2)} \frac{1}{(1+\bar{\lambda}_{xx})(1+\bar{\rho}_{yy})} \left[\bar{\lambda}_{xx} \bar{e}_x^2 \frac{\partial^8 w^1}{\partial x^8} + \Lambda_2 \frac{\partial^8 w^1}{\partial x^6 \partial y^2} \right. \\
&+ \Lambda_3 \frac{\partial^8 w^1}{\partial x^4 \partial y^4} + \Lambda_4 \frac{\partial^8 w^1}{\partial x^2 \partial y^6} + \bar{e}_y^2 \bar{\lambda}_{yy} \frac{\partial^8 w^1}{\partial y^8} + 2\bar{e}_x \left(\frac{\pi}{L}\right)^2 \bar{\lambda}_{xx} \frac{\partial^6 w^1}{\partial x^6}
\end{aligned}$$

$$\begin{aligned}
& - 2\left(\frac{\pi}{L}\right)^2 \Lambda_5 \frac{\partial^6 w^1}{\partial x^4 \partial y^2} + 2\left(\frac{\pi}{L}\right)^2 \bar{v} \bar{e}_y \lambda_{yy} \frac{\partial^6 w^1}{\partial x^2 \partial y^4} + \left(\frac{\pi}{L}\right)^4 \Lambda_6 \frac{\partial^4 w^1}{\partial x^4} \Big/ \\
& \left[\frac{\partial^4 w^1}{\partial x^4} + \frac{\Lambda_1}{1+\bar{\lambda}_{xx}} \frac{\partial^4 w^1}{\partial x^2 \partial y^2} + \frac{1+\bar{\lambda}_{yy}}{1+\bar{\lambda}_{xx}} \frac{\partial^4 w^1}{\partial y^4} \right] \\
& - \bar{K}_{yy} \left(\frac{L}{\pi}\right)^2 \left(\frac{L}{\pi R}\right)^2 \frac{1}{1+\bar{\rho}_{yy}} \left[\bar{e}_x \frac{\bar{\lambda}_{xx}}{1+\bar{\lambda}_{xx}} \frac{\partial^6 w^1}{\partial x^6} + \Lambda_8 \frac{\partial^6 w^1}{\partial x^4 \partial y^2} \right. \\
& + \Lambda_9 \frac{\partial^6 w^1}{\partial x^2 \partial y^4} + \bar{e}_y \frac{\bar{\lambda}_{yy}}{1+\bar{\lambda}_{xx}} \frac{\partial^6 w^1}{\partial y^6} - \left(\frac{\pi}{L}\right)^2 \frac{\nu}{1+\bar{\lambda}_{xx}} \frac{\partial^4 w^1}{\partial x^4} \\
& + \left(\frac{\pi}{L}\right)^2 \Lambda_{10} \frac{\partial^4 w^1}{\partial x^2 \partial y^2} - \left(\frac{\pi}{L}\right)^2 \frac{1+\bar{\lambda}_{yy}}{1+\bar{\lambda}_{xx}} \frac{\partial^4 w^1}{\partial y^4} \Big] \Big/ \left[\frac{\partial^4 w^1}{\partial x^4} \right. \\
& + \frac{\Lambda_1}{1+\bar{\lambda}_{xx}} \frac{\partial^4 w^1}{\partial x^2 \partial y^2} + \frac{1+\bar{\lambda}_{yy}}{1+\bar{\lambda}_{xx}} \frac{\partial^4 w^1}{\partial y^4} \Big] - \left(\frac{L}{\pi}\right)^2 \frac{1}{1+\bar{\rho}_{yy}} \left[\left(\frac{1}{2} \bar{K}_{yy} \right. \right. \\
& \left. \left. - \bar{K}_{xx} \right) \frac{\partial^2 w^1}{\partial x^2} + \bar{K}_{yy} \frac{\partial^2 w^1}{\partial y^2} + 2\bar{K}_s \frac{\partial^2 w^1}{\partial x \partial y} \right] = 0 \tag{6}
\end{aligned}$$

It should be noted that for a pure torsional loading $\bar{K}_{xx} = 0 = \bar{K}_{yy}$ and for the loading case of torsion combined with axial compression, $\bar{K}_{yy} = 0$.

Since no closed form solution is possible for the above differential equation, the Galerkin procedure is employed. The series used (Reference [28]) in the procedure is given by

$$w_{\text{appr.}}^1 = \sum_{m=1}^{\infty} \left[A_m \sin \frac{n y}{R} + B_m \cos \frac{n y}{R} \right] \sin \frac{m \pi x}{L} \tag{7}$$

No summation of n is required because of uncoupling of the terms. For details on satisfaction of boundary conditions see References [28] and [42]. For the error to be a minimum the following equations must be satisfied (Galerkin Integrals)

$$\int_0^L \int_0^{2\pi R} [L(w_{\text{appr.}}^1)] \begin{Bmatrix} \sin \frac{n'y}{R} \\ \cos \frac{n'y}{R} \end{Bmatrix} \sin \frac{m'\pi x}{L} dy dx = 0 \quad (8)$$

where $m' = 1, 2, \dots, \infty$.

Substitution from Equations (6) and (7) into Equation (8) and execution of the required integrations leads to the following characteristic equation (only up to 7th order shown herein).

$$\begin{matrix} m' \\ m \end{matrix} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{matrix} \begin{bmatrix} \gamma_1 & \frac{2}{3}\bar{K}_s & 0 & \frac{4}{15}\bar{K}_s & 0 & \frac{6}{35}\bar{K}_s & 0 \\ \frac{2}{3}\bar{K}_s & \gamma_2 & -\frac{6}{5}\bar{K}_s & 0 & -\frac{10}{21}\bar{K}_s & 0 & -\frac{14}{45}\bar{K}_s \\ 0 & -\frac{6}{5}\bar{K}_s & \gamma_3 & \frac{12}{7}\bar{K}_s & 0 & \frac{2}{3}\bar{K}_s & 0 \\ \frac{4}{15}\bar{K}_s & 0 & \frac{12}{7}\bar{K}_s & \gamma_4 & -\frac{20}{9}\bar{K}_s & 0 & -\frac{28}{33}\bar{K}_s \\ 0 & -\frac{10}{21}\bar{K}_s & 0 & -\frac{20}{9}\bar{K}_s & \gamma_5 & \frac{30}{11}\bar{K}_s & 0 \\ \frac{6}{35}\bar{K}_s & 0 & \frac{2}{3}\bar{K}_s & 0 & \frac{30}{11}\bar{K}_s & \gamma_6 & -\frac{42}{13}\bar{K}_s \\ 0 & -\frac{14}{45}\bar{K}_s & 0 & -\frac{28}{33}\bar{K}_s & 0 & -\frac{42}{13}\bar{K}_s & \gamma_7 \end{bmatrix} = 0 \quad (9)$$

where

$$\gamma_m = \frac{\pi}{8\beta} \left[\Lambda_{11} + \frac{12Z^2}{\pi^4(1-\nu^2)} \frac{\Lambda_{12}}{\Lambda_{13}} - \bar{K}_{xx}(m^2 - \xi\Lambda_{14}) \right] \quad (10)$$

The new constants introduced in Equation (10) are given by

$$\begin{aligned} \Lambda_{11} &= (1+\bar{\rho}_{xx})m^4 + 2\Lambda_7 m^2 \beta^2 + (1+\bar{\rho}_{yy})\beta^4 \\ \Lambda_{12} &= \bar{\lambda}_{xx} \bar{e}_x^2 m^8 + \Lambda_2 m^6 \beta^2 + \Lambda_3 m^4 \beta^4 + \Lambda_4 m^2 \beta^6 + \bar{e}_y^2 \bar{\lambda}_{yy} \beta^8 \\ &\quad - 2\nu \bar{e}_x \bar{\lambda}_{xx} m^6 + 2\Lambda_5 m^4 \beta^2 - 2\nu \bar{e}_y \bar{\lambda}_{yy} m^2 \beta^4 + \Lambda_6 m^4 \\ \Lambda_{13} &= (1+\bar{\lambda}_{xx})m^4 + \Lambda_1 m^2 \beta^2 + (1+\bar{\lambda}_{yy})\beta^4 \end{aligned} \quad (11)$$

$$\begin{aligned} \Lambda_{14} &= \left(\frac{L}{\pi R} \right)^2 \left[\bar{e}_x \frac{\bar{\lambda}_{xx}}{1+\bar{\lambda}_{xx}} m^6 + \Lambda_8 m^4 \beta^2 + \Lambda_9 m^2 \beta^4 \right. \\ &\quad \left. + \bar{e}_y \frac{\bar{\lambda}_{yy}}{1+\bar{\lambda}_{xx}} \beta^6 + m^4 \frac{\nu}{1+\bar{\lambda}_{xx}} - \Lambda_{10} m^2 \beta^2 + \frac{1+\bar{\lambda}_{yy}}{1+\bar{\lambda}_{xx}} \beta^4 \right] / \\ &\quad \left[m^4 + \frac{\Lambda_1}{1+\bar{\lambda}_{xx}} m^2 \beta^2 + \frac{1+\bar{\lambda}_{yy}}{1+\bar{\lambda}_{xx}} \beta^4 \right] + \frac{1}{2} m^2 + \beta^2 \end{aligned}$$

where ξ is the ratio of pressure to axial load coefficients, $\bar{K}_{yy}/\bar{K}_{xx}$, $\beta = nL/\pi R$, and $m = 1, 2, 3, \dots, \infty$.

Solving Equation (9) for \bar{K}_s and minimizing w.r.t. β , so that $\beta \geq 2L/\pi R^*$, the critical load coefficient, $\bar{K}_{s_{cr}}$ is obtained for any

*It should be noted that $n = 0$ is not possible as it does not give any deflection mode in the circumferential direction [see Equation (7)]. Also n cannot be equal to one since it represents a rigid body displacement. Hence n must be greater than or equal to 2.

given shell geometry. This requires one dimensional search for $\bar{K}_{s_{cr}}$. The higher the order of the determinant, the better the solution converges to the classical general instability buckling load, but at the same time the computer time increases severely. On this point, care has been taken to consider the least possible order of determinant in order to get the best possible results for each loading case.

For a pure torsional load, the last term with \bar{K}_{xx} in Equation (10) is zero. For torsion combined with axial compression, \bar{K}_{xx} has a specified value but $\xi = 0$. If all the three types of loads are present then the known (applied loads) values of \bar{K}_{xx} and ξ are substituted in Equation (10).

Panel Instability Critical Load

The panel instability critical load coefficient, $\bar{K}_{sp_{cr}}$, for any given geometry, is obtained by setting all ring parameters equal to zero, i.e.

$$\begin{aligned}\bar{\lambda}_{yy} &= 0, & \bar{\rho}_{yy} &= 0 \\ \bar{e}_y &= 0\end{aligned}$$

letting $L = l_y$, solving the determinant, Equation (9), for \bar{K}_s , and by minimizing it w.r.t. β so that $\beta \geq 2L/\pi R$. Once again, a one-dimensional search is required. It should be noted that the skin thickness, h , remains the same as used for general instability. However, $Z\left(= \frac{L^2(1-\nu^2)^{1/2}}{Rh}\right)$ changes because of changed value of L . The critical uniform torsional stress resultant for panel instability, $\bar{N}_{xyp_{cr}}$ is obtained from the basic definition of \bar{K}_s so that

$$\bar{N}_{xyp_{cr}} = \bar{K}_{sp_{cr}} \frac{\pi^2 D}{L^2} \quad (12)$$

Prebuckling Stresses

With the assumption that only the skin takes the membrane shear force, the stress resultants are related to the strains by the relations

$$\begin{aligned} N_{xx}^o &= \frac{Eh}{1-\nu^2} (\epsilon_{xx}^o + \nu \epsilon_{yy}^o) + \frac{E_x A_x}{l_x} \epsilon_{xx}^o \\ N_{yy}^o &= \frac{Eh}{1-\nu^2} (\epsilon_{yy}^o + \nu \epsilon_{xx}^o) + \frac{E_y A_y}{l_y} \epsilon_{yy}^o \\ N_{xy}^o &= \frac{Eh}{2(1+\nu)} \gamma_{xy}^o (= \sigma_{xy_{sk}} h) \end{aligned} \quad (13)$$

where the superscript "o" represents the prebuckling parameters. Also from a statical solution

$$\begin{aligned} N_{xx}^o &= \frac{qR}{2} - \bar{N} \\ N_{yy}^o &= qR \\ N_{xy}^o &= \frac{T}{2\pi R^2} \end{aligned} \quad (14)$$

Substituting

$$\frac{A_x}{l_x} = \frac{Eh\bar{\lambda}_{xx}}{E_x(1-\nu^2)} \quad \text{and} \quad \frac{A_y}{l_y} = \frac{Eh\bar{\lambda}_{yy}}{E_y(1-\nu^2)}$$

from Equation (5) and N_{xx}^0 , N_{yy}^0 , N_{xy}^0 from Equation (14) into Equation (13), putting $\xi = \frac{qR}{\bar{N}}$ ($= \bar{K}_{yy}/\bar{K}_{xx}$) and solving for ϵ_{xx}^0 , ϵ_{yy}^0 , one easily obtains

$$\epsilon_{xx}^0 = \left(\frac{1-\nu^2}{Eh} \right) \bar{N} \frac{\left(\frac{\xi}{2} - 1 \right) (1 + \bar{\lambda}_{yy}) - \nu \xi}{(1 + \bar{\lambda}_{xx})(1 + \bar{\lambda}_{yy}) - \nu^2}$$

$$\epsilon_{yy}^0 = \left(\frac{1-\nu^2}{Eh} \right) \bar{N} \frac{\xi(1 + \bar{\lambda}_{xx}) - \nu \left(\frac{\xi}{2} - 1 \right)}{(1 + \bar{\lambda}_{xx})(1 + \bar{\lambda}_{yy}) - \nu^2}$$
(15)

Now the stresses in the skin and the stiffeners are given by

$$\sigma_{xx_{sk}} = \left(\frac{E}{1-\nu^2} \right) (\epsilon_{xx}^0 + \nu \epsilon_{yy}^0)$$

$$\sigma_{yy_{sk}} = \left(\frac{E}{1-\nu^2} \right) (\epsilon_{yy}^0 + \nu \epsilon_{xx}^0)$$
(16)

$$\sigma_{xy_{sk}} = G_{xy} \gamma_{xy}^0$$

$$\sigma_{xx_{st}} = E_x \epsilon_{xx}^0$$

$$\sigma_{yy_r} = E_y \epsilon_{yy}^0$$

From Equations (13), (15) and (16), the prebuckling stresses in the skin, stringers and rings are given by

Stresses in Skin

$$\sigma_{xx_{sk}} = \frac{\bar{N}}{h} \frac{\left(\frac{\xi}{2} - 1\right)(1 + \bar{\lambda}_{yy} - \nu^2) + \nu\xi\bar{\lambda}_{xx}}{(1 + \bar{\lambda}_{xx})(1 + \bar{\lambda}_{yy}) - \nu^2}$$

$$\sigma_{yy_{sk}} = \frac{\bar{N}}{h} \frac{\xi(1 + \bar{\lambda}_{xx}) - \xi\nu^2 + \nu\bar{\lambda}_{yy}\left(\frac{\xi}{2} - 1\right)}{(1 + \bar{\lambda}_{xx})(1 + \bar{\lambda}_{yy}) - \nu^2} \quad (17a)$$

$$\sigma_{xy_{sk}} = \frac{\bar{N}_{xy}}{h}$$

Stress in Stringers

$$\sigma_{xx_{st}} = \frac{E_x(1-\nu^2)}{Eh} \bar{N} \frac{\left(\frac{\xi}{2} - 1\right)(1 + \bar{\lambda}_{yy}) - \nu\xi}{(1 + \bar{\lambda}_{xx})(1 + \bar{\lambda}_{yy}) - \nu^2} \quad (17b)$$

Stress in Rings

$$\sigma_{yy_r} = \frac{E_y(1-\nu^2)}{Eh} \bar{N} \frac{\xi(1 + \bar{\lambda}_{xx}) - \nu\left(\frac{\xi}{2} - 1\right)}{(1 + \bar{\lambda}_{xx})(1 + \bar{\lambda}_{yy}) - \nu^2} \quad (17c)$$

It should be noted that for a pure torsion case all stresses are zero except $\sigma_{xy_{sk}}$.

Local Buckling of Stringers and Rings

For a pure torsional load there are no stresses either in the stringers or in the rings and therefore there is no stiffener buckling.

For the loading case torsion combined with axial compression with or without (internal) lateral pressure and internal stiffening, as

considered in this work, the rings are always in tension and so, once again, there is no possibility of ring buckling failure. Hence, the buckling failure of rings does not exist for the system considered. However, if one has to consider external pressure and/or outside stiffening, the rings could be in compression and one should refer to [26] for finding the critical stresses in the rings.

In the case of combined load case, mentioned above, the stringers are under compression and their buckling failure has to be considered. The stringers are treated as collection of flat plates. For all open type stringers excepting rectangular, when the rings are deeper than the stringers, the portion of the stringer between any two adjacent rings is treated as a rectangular flat plate of length ℓ_y , the web considered as four edges simply supported and the flange as three edges simply supported and the fourth edge free. When the stringers are deeper than the rings, the portion of the stringer web up to the ring depth is considered as a rectangular flat plate of length ℓ_y with all four edges simply supported, and the outstanding portion of the web is treated as a rectangular flat plate of length L with the four edges simply supported. The flanges are treated as rectangular plates with three edges simply supported and one edge free. There being no flanges, the outstanding portion of rectangular stringers are treated as three edges simply supported and the fourth edge free having length L . When the rings are deeper, then a rectangular stringer is treated as a plate of length ℓ_y with three edges simply supported and the fourth edge free. All the edges of a hat stringer are treated as simply supported. It was observed, for the combined load case and the designs

considered, that because of either the stress becoming higher than the yield stress or the consideration of minimum gauge, the stringer depth was always less than the ring depth.

The critical stresses for various types of stringers used in this work are found from Bleich [29] and Bulson [30] for the case when the rings are deeper than the stringers and are given in Table 1. The dimensional proportionalities of the stringers are shown in Figure B1 of Appendix B.

Skin Wrinkling

Pure Torsion

For a pure torsional load the prebuckling stress in the skin is given by

$$\sigma_{xy_{sk}} = \frac{\bar{N}_{xy}}{h}.$$

Assuming all the supported edges of the skin as simply supported and treating it as a flat rectangular plate the local buckling stress, from Reference [29], is given by

$$\sigma_{xy_{sk_{cr}}} = \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{h}{b}\right) \left(5.34 + \frac{4}{\alpha^2}\right) \quad (18)$$

where $\alpha = a/b =$ aspect ratio, 'a' standing for the bigger dimension and 'b' the smaller dimension out of l_x and l_y .

Torsion Combined with Axial Compression (without Lateral Pressure)

For this type of loading, the prebuckling stresses in the skin

Table 1. Critical Stresses in Stringers.

Stringer Type	$\sigma_{xxstw_{cr}}$	$\sigma_{xxstw_{fcr}}$
RS	$\frac{\pi^2 E_x}{12(1-\nu^2)} \left(\frac{t_x}{d_x} \right)^2 \left[\left(\frac{d_x}{l_y} \right)^2 + \frac{6(1-\nu)}{\pi^2} \right]$	----
TS	$\frac{\pi^2 E_x}{3(1-\nu^2)} \left(\frac{t_{wx}}{d_{wx}} \right)^{2*}$	$\frac{\pi^2 E_x}{12(1-\nu^2)} \left(\frac{2t_{fx}}{b_{fx}-t_{wx}} \right)^2 \left[\left(\frac{b_{fx}-t_{wx}}{2l_y} \right)^2 + \frac{6(1-\nu)}{\pi^2} \right]$
IAS	$\frac{\pi^2 E_x}{3(1-\nu^2)} \left(\frac{t_{wx}}{d_{wx}} \right)^{2*}$	$\frac{\pi^2 E_x}{12(1-\nu^2)} \left(\frac{t_{fx}}{b_{fx}-t_{wx}} \right)^2 \left[\left(\frac{b_{fx}-t_{wx}}{l_y} \right)^2 + \frac{6(1-\nu)}{\pi^2} \right]$
CS,ZS	$\frac{\pi^2 E_x}{3(1-\nu^2)} \left(\frac{t_{wx}}{d_{wx}-2t_{fx}} \right)^{2*}$	$\frac{\pi^2 E_x}{12(1-\nu^2)} \left(\frac{t_{fx}}{b_{fx}} \right)^2 \left[\left(\frac{b_{fx}}{l_y} \right)^2 + \frac{6(1-\nu)}{\pi^2} \right]$

(Continued)

Table 1. Critical Stresses in Stringers. (Continued)

Stringer Type	$\sigma_{xxstw_{cr}}$	$\sigma_{xxstw_{fcr}}$
IS	$\frac{\pi^2 E_x}{3(1-\nu^2)} \left(\frac{t_{wx}}{d_{wx} - 2t_{fx}} \right)^{2*}$	$\frac{\pi^2 E_x}{12(1-\nu^2)} \left(\frac{2t_{fx}}{b_{fx}} \right)^2 \left[\left(\frac{b_{fx}}{2l_y} \right)^2 + \frac{6(1-\nu)}{\pi^2} \right]$
AS	$\frac{\pi^2 E_x}{12(1-\nu^2)} \left(\frac{t_{wx}}{d_{wx} - t_{fx}} \right)^2 \left[\left(\frac{d_{wx} - t_{fx}}{l_y} \right)^2 + \frac{6(1-\nu)}{\pi^2} \right]$	----
HS	$\frac{\pi^2 E_x}{3(1-\nu^2)} \left(\frac{t_{1x}}{d_x - t_{3x}} \right)^{2*}$	$\frac{\pi^2 E_x}{3(1-\nu^2)} \left(\frac{t_{2x}}{k_{1x} d_x} \right)^2$

* In the case of design without minimum gauge, it is possible that $l_y/d_{wx} < 1$ and then the web will behave as a short plate. If this be so, then the critical buckling stress in the web will have the form: $\sigma_{xxstw_{cr}} = \frac{\pi^2 E_x}{12(1-\nu^2)} \left(\frac{a}{b} \right)^2 \left(\frac{b}{l_y} + \frac{l_y}{b} \right)^2$. For example, for IS,

$$\sigma_{xxstw_{cr}} = \frac{\pi^2 E_x}{12(1-\nu^2)} \left(\frac{t_{wx}}{d_{wx} - 2t_{fx}} \right)^2 \left(\frac{d_{wx} - 2t_{fx}}{l_y} + \frac{l_y}{d_{wx} - 2t_{fx}} \right)^2$$

are given by Equation (17a). Again treating the skin as a flat rectangular plate with all four edges simply supported and neglecting the small value $\sigma_{yy_{sk}}$ compared to $\sigma_{xx_{sk}}$ and $\sigma_{xy_{sk}}$, the critical stress in the skin, from Reference [29] is given below.

(a) For $\alpha (= l_y/l_x) \geq 1$ (Long plate):

$$\sigma_{xy_{sk_{cr}}} = \frac{\pi^2 E}{6(1-\nu^2)} \left(\frac{h}{b}\right)^2 \eta_k^2 \left(-1 + \sqrt{1 + \frac{4}{\eta_k^2}}\right) \quad (19a)$$

$$\sigma_{xx_{sk_{cr}}} = \eta \sigma_{xy_{sk_{cr}}}$$

where

$$k = \frac{4}{3} + \frac{1}{\alpha^2}$$

(b) For $1/2 < \alpha (= l_y/l_x) < 1$ (short plate):

$$\sigma_{xy_{sk_{cr}}} = \frac{\pi^2 E}{24(1-\nu^2)} \left(\frac{h}{b}\right)^2 \left(\alpha + \frac{1}{\alpha}\right)^2 \eta_k^2 \left(-1 + \sqrt{1 + \frac{4}{\eta_k^2}}\right) \quad (19b)$$

$$\sigma_{xx_{sk_{cr}}} = \eta \sigma_{xy_{sk_{cr}}}$$

where

$$k = \frac{4\alpha^2 + 5.34}{(\alpha^2 + 1)^2}$$

For (a) as well as (b),

$$\eta = \sigma_{xx_{sk}} / \sigma_{xy_{sk}}$$

Torsion Combined with Axial Compression and Lateral Pressure (internal)

Once again for this loading case, the prebuckling stresses in the skin are given by Equation (17a), but $\sigma_{yy_{sk}}$ is no more negligible compared to $\sigma_{xx_{sk}}$ and $\sigma_{xy_{sk}}$. Hence $\sigma_{xy_{sk_{cr}}}$ is found by the program developed in Reference [31] by considering the skin as a circular cylindrical panel of length l_y and width l_x with all four edges simply supported and substituting all the stringer and ring parameters equal to zero, i.e.

$$\bar{\lambda}_{xx} = 0 ; \quad \bar{\lambda}_{yy} = 0$$

$$\bar{\rho}_{xx} = 0 ; \quad \bar{\rho}_{yy} = 0$$

$$\bar{e}_x = 0 ; \quad \bar{e}_y = 0$$

R and h are known already but Z has a different value for different l_y .

Von Mises-Hencky Yield Criterion for the Skin

During the combined loading the skin is subjected to σ_{xx} , σ_{yy} and σ_{xy} . If k is the yield stress in pure shear then applying the von Mises-Hencky yield criterion, the second invariant of the deviatoric part of stress in the skin, σ_s is given by

$$\sigma_s = \left(\frac{\sigma_{xx}^2 + \sigma_{yy}^2 - \sigma_{xx}\sigma_{yy}}{3} + \sigma_{xy}^2 \right)^{1/2} \quad (20)$$

σ_s must be less than k and the ratio σ_s/k gives the skin yielding coefficient, SY, which must be within allowable limits.

Formulation of the Objective Function and Design Variables

Neglecting the common stiffener materials at the stringer-ring

intersections, the weight of the composite shell is given by

$$W = 2\pi RLh\rho_{sk} + \rho_x \int_0^L \int_0^{2\pi R} \frac{A_x}{l_x} dydx + \rho_y \int_0^L \int_0^{2\pi R} \frac{A_y}{l_y} dydx \quad (21)$$

Substituting A_x/l_x and A_y/l_y from Equation (5) and performing the integrations

$$W = 2\pi RLh\rho_{sk} \left[1 + \frac{1}{1-\nu^2} \left(\frac{E\rho_x \bar{\lambda}_{xx}}{E_x \rho_{sk}} + \frac{E\rho_y \bar{\lambda}_{yy}}{E_y \rho_{sk}} \right) \right] \quad (22)$$

which, for the case when $E = E_x = E_y$ and $\rho_{sk} = \rho_x = \rho_y$, becomes

$$W = 2\pi RLh\rho_{sk} \left[1 + \frac{\bar{\lambda}_{xx} + \bar{\lambda}_{yy}}{1-\nu^2} \right] \quad (22a)$$

In Equation (22), the three terms represent the individual weights of the skin, stringer and ring materials respectively.

Based on the premise that general instability is one of the active mode of failure, W is minimized subject to the equality constraint that $\bar{N}_{xy_{cr}}$ should have a specified value \bar{N}_{xy} . Thus, through the use of penalty parameter the augmented objective function is

$$W^* = W + \lambda |\bar{N}_{xy_{cr}} - \bar{N}_{xy}| \quad (23)$$

where λ is penalty parameter. \bar{N}_{xy} includes a factor of safety and knockdown factor.

Substituting for

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$$\bar{N}_{xy\text{cr}} = \frac{\pi^2 D}{L^2} \bar{K}_{s\text{cr}} = \frac{\pi^2 E h^3}{12(1-\nu^2)L^2} \bar{K}_{s\text{cr}}$$

from Equation (5) and W from Equation (22) in Equation (23) and recalling that $Rh/L^2(1-\nu^2) = 1/Z$, (after completing the algebraic manipulations) the nondimensionalized form of the objective function becomes

$$\bar{W}^* = \bar{W} + \lambda^* |\bar{K}_{s\text{cr}}^* - Z \bar{N}_{xy}^*| \quad (24)$$

where

$$\begin{aligned} \bar{W}^* &= \frac{ZW^*}{2\pi L^3 \rho_{sk} (1-\nu^2)^{1/2}} \\ \bar{W} &= 1 + \frac{1}{1-\nu^2} \left(\frac{E \rho_x \bar{\lambda}_{xx}}{E_x \rho_{sk}} + \frac{E \rho_y \bar{\lambda}_{yy}}{E_y \rho_{sk}} \right) \\ \lambda^* &= \frac{\lambda \pi L E}{24 \rho_{sk} R^3} \\ \bar{K}_{s\text{cr}}^* &= \frac{\bar{K}_{s\text{cr}}}{Z^2} \\ \bar{N}_{xy}^* &= \frac{12 R^3 \bar{N}_{xy}}{\pi E L^4 (1-\nu^2)^{1/2}} \end{aligned} \quad (25)$$

L , R , \bar{N}_{xy} , ν , E 's and ρ 's are fixed for a particular design problem from practical considerations. Furthermore, starting with the definitions given in Equation (5), \bar{e}_x , \bar{e}_y , $\bar{\rho}_{xx}$ and $\bar{\rho}_{yy}$ are expressed

in terms of Z , $\bar{\lambda}_{xx}$, $\bar{\lambda}_{yy}$, nondimensionalized radii of gyration $\bar{\alpha}_x$, $\bar{\alpha}_y$ and shape parameters C_x and C_y for all types of stiffeners as shown below. By doing this not only the number of independent variables are reduced to a minimum but also the resulting design parameters, $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$, even though unbounded, have a shorter range.

$$\bar{e}_x = \frac{\pi^2(1-\nu^2)^{1/2}}{2Z} (1 + C_x \bar{\alpha}_x) ; \quad \bar{e}_y = \frac{\pi^2(1-\nu^2)^{1/2}}{2Z} (1 + C_y \bar{\alpha}_y) \quad (26)$$

$$\bar{\rho}_{xx} = \bar{\alpha}_x^{-2} \bar{\lambda}_{xx} \quad ; \quad \bar{\rho}_{yy} = \bar{\alpha}_y^{-2} \bar{\lambda}_{yy}$$

where $\bar{\alpha}_x = p_x(d_x/h)$ and $\bar{\alpha}_y = p_y(d_y/h)$, p_x and p_y being the stringer and ring shape multipliers for their radii of gyration. Also, recalling that the torsional rigidity J for a closed type of stiffener is given by

$$J = \frac{4A^2}{\int_0^s \frac{ds}{t}} \quad (27)$$

where A = total area enclosed by the center line of the wall, t is the thickness and s is the length of the center line of the wall cross section, one can easily show that

$$\frac{(GJ)_x}{2D\ell_x} + \frac{(GJ)_y}{2D\ell_y} = q_x \bar{\lambda}_{xx} \bar{\alpha}_x^{-2} + q_y \bar{\lambda}_{yy} \bar{\alpha}_y^{-2} \quad (28)$$

q_x and q_y being the stringer and ring shape multipliers for their torsional rigidities. The expressions for C_x , C_y , p_x , p_y and q_x , q_y for different stiffener shapes are shown in Appendix B for the opened

and closed type of stiffeners used herein.

It can easily be seen that $C_x, C_y, p_x, p_y, q_x, q_y$ have definite numerical values depending upon the geometric proportionalities of the stiffeners used. In addition, β is fixed from the fact that it must have a definite value to yield $\bar{K}_{s_{cr}}$. Hence \bar{W}^* is given by

$$\bar{W}^* = \bar{W}^*(Z, \bar{\lambda}_{xx}, \bar{\lambda}_{yy}, \bar{\alpha}_x, \bar{\alpha}_y) \quad (29)$$

Equation (24) is derived from Equation (23). A simple comparison shows that as $\lambda^* \rightarrow \infty$ the unconstrained minimization of \bar{W}^* approaches the constrained minimization of \bar{W} with $\bar{K}_{s_{cr}}^* = Z \bar{N}_{xy}^*$, which implies the constrained minimization of W with $\bar{N}_{xy_{cr}} = \bar{N}_{xy}^*$, as desired. Practical limitations on d_x and d_y always put upper bounds on $\bar{\alpha}_x$ and $\bar{\alpha}_y$. From a MG consideration on the skin thickness, h or otherwise the bounds on Z can be easily fixed. So for a fixed and given value of \bar{N}_{xy}^* , \bar{W}^* is a function of five independent variables. The method adopted in this work is to fix $\bar{\alpha}_x, \bar{\alpha}_y$ and Z , substitute the known value of \bar{N}_{xy}^* and minimize \bar{W}^* w.r.t. the two discrete variables $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$. The search techniques, solution procedures, and the design steps employed for each loading case so as to yield the minimum weight design satisfying all the constraints with various types of stiffeners, are given in the next chapter.

CHAPTER III

MINIMUM WEIGHT DESIGN SOLUTION PROCEDURES

As shown in the previous chapter, for a pure torsional type of load the prebuckling stress in the skin is pure shear, and the stiffeners are stress-free. However, for a combined load case with or without lateral pressure the skin is subjected to normal stresses in the direction of the x-y axes associated with a shear stress, and the stringers and rings are subjected to unidimensional compression and tension respectively. Besides, a minimum weight design for a combined load case demands an essential precaution to be taken care of. One has to be careful to see that the shell does not buckle with a single load at any stage of optimization. If the shell does buckle with a single load the solution thus obtained will not be the one sought for.

Due to reasons stated above the solution procedures for the combined load case are quite different from that of pure torsion case. These are given below in detail.

Pure Torsion

The minimum weight design is obtained in two stages. In Stage 1 the range of Z-values is found (see Appendix C) and the upper bounds on $\bar{\alpha}_x$ and $\bar{\alpha}_y$ are fixed from practical considerations on d_x and d_y . For a fixed value of Z, an unconstrained minimization of \bar{W}^* , see Equation (24), is performed w.r.t. $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$ in $\bar{\alpha}_x - \bar{\alpha}_y$ space by using an optimization technique. The data generated are recorded. In Stage 2 these data

are used to design a shell that satisfies all other constraints. Satisfaction of these constraints requires that the prebuckling stress in the skin, as given by Equation (17a), must be less than the yield stress in shear for the skin material so that yielding of the skin does not take place; the critical stress in the skin as given by Equation (18) must be greater than the prebuckling stress found from Equation (17a); the panel buckling load \bar{N}_{xy_p} must be greater than the general instability load \bar{N}_{xy} for panel buckling not to take place; and finally simultaneous failure modes should be avoided. Furthermore, if a MG (minimum gauge) design is desired then all the dimensions must be greater than or equal to the prescribed value.

Stage 1: Optimization and Data Generation

In order to minimize \bar{W}^* w.r.t. $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$ at fixed Z within the bounds of $\bar{\alpha}_x$ and $\bar{\alpha}_y$ decided upon, flexible polyhedron type of simplex method (Reference [32]) is used. This two-dimensional search technique is used because, as proved by Box [33] and supported by Himmelblau [34], Siddall [35] and Dixon [36], when the function to be minimized is highly nonlinear, as in this case, it is the most efficient by comparison to all the direct (derivative-free) search methods available at present. The reflection, contraction and expansion coefficients (α , k and γ) are finally adopted equal to 1.0, 0.5 and 2.0, respectively, after trying a number of values and observing the rate of convergence. Incidentally the same values have been recommended in References [32] and [34]. It has been shown in Reference [37] that when λ^* is sufficiently large then the unconstrained minimization of \bar{W}^* approaches the constrained minimization of \bar{W} under the constraint $\bar{K}_{s_{cr}}^* = Z\bar{N}_{xy}^*$, the exact solution being

obtained when λ^* approaches infinity. The value of λ^* equal to 10^6 is fixed by selecting different values for typical set of parameters and observing the difference between \bar{W}^* and \bar{W} and the number of iterations required for convergence. Lower value of λ^* will not ensure convergence [37]. With too high a value for λ^* the computer time increases. The minimum value of \bar{W} as determined above is denoted as \bar{W}^{\sim} .

Because of the complexity of the function it is not feasible to prove analytically that the function \bar{W}^* and the penalty function are convex. However, in order to ensure obtaining a true minimum, different starting points are tried for selected set of parameters.

The above procedure requires the evaluation of $\bar{K}_{s_{cr}}$ at each vertex of the simplex. This is obtained by solving the determinant, Equation (9) for \bar{K}_s and minimizing \bar{K}_s w.r.t. β ($= nL/\pi R$, n being greater than or equal to 2). The higher the order of the determinant, the better the solution converges to the exact value. It was found by Simitses [28] and corroborated in this work, that the convergence is rapid and it is sufficient to consider a fifth order determinant. So the fifth order determinant is expanded to yield a quadratic equation in \bar{K}_s^2 which is solved for \bar{K}_s . The value of \bar{K}_s , thus obtained, is minimized w.r.t. β by the one dimensional golden section search technique, assuming that β is a continuous variable [38].

Then \bar{K}_s is evaluated for two integer values of n , one on each side of the noninteger value of n as determined by the golden section technique, and the one corresponding to the smaller of those two values of \bar{K}_s is selected as $\bar{K}_{s_{cr}^{\sim}}$.

A typical plot of \bar{W} is shown in Figure 3.

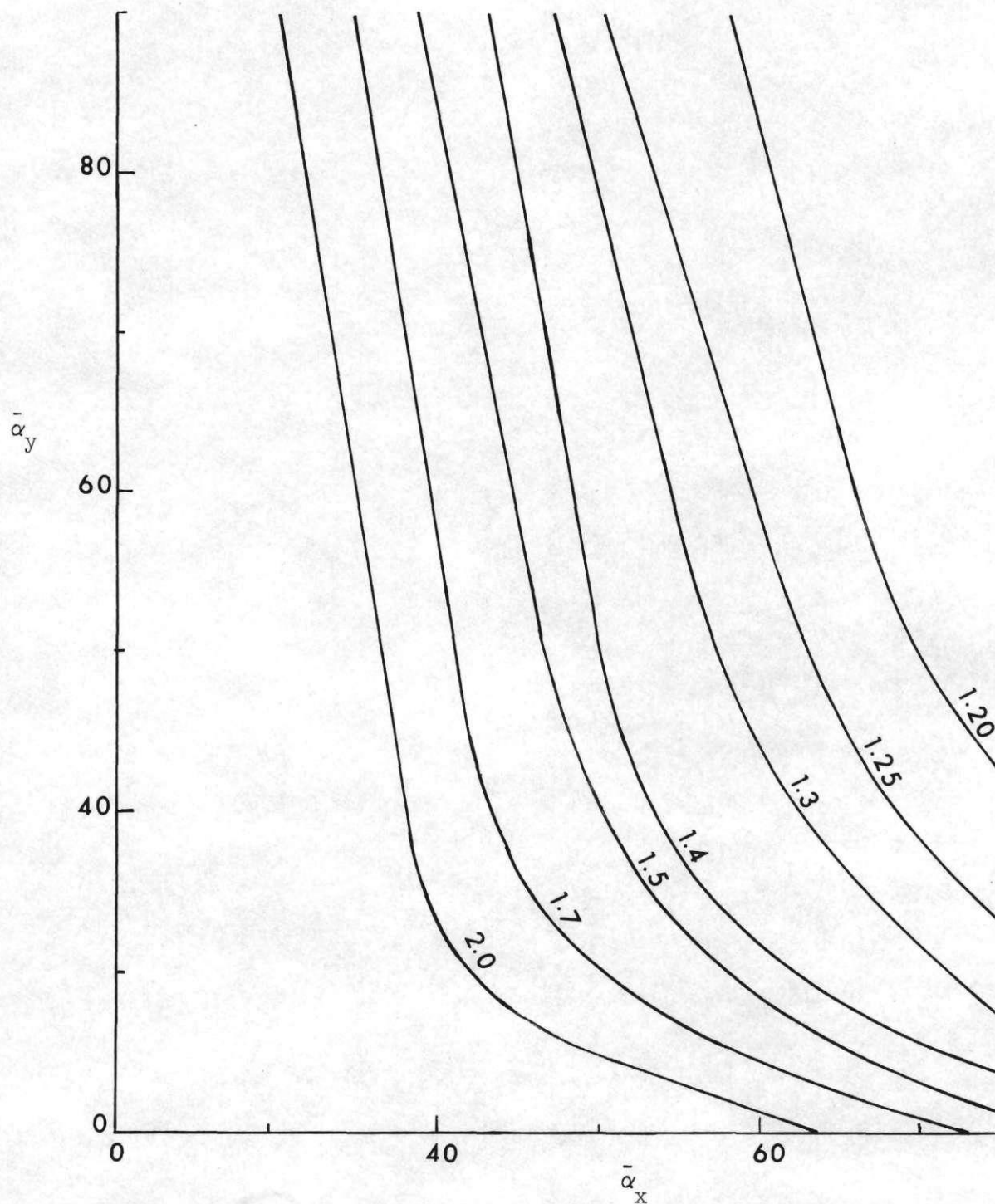


Figure 3: Design Chart for \tilde{W} with RS-RR ($Z=2,000$)

Stage 2: Design

The steps required for a minimum weight design with different types of stiffener geometries and data generated from Stage 1 are listed below.

1. For the values of Z selected for data generation, find $\sigma_{xy_{sk}}$ from Equation (17) and see that this value is less than the yield stress in shear for the skin material. If the skin has yielded, then select a smaller value of Z . Start design with the highest Z -value first by adopting the following steps.

2. Determine the minimum number of rings (3 taken in this work [41]) and/or stringers from a consideration of applicability of the smeared technique. This will fix higher limits on l_y and l_x . Note that this step must also include the possibility of producing only a stringer-stiffened or ring-stiffened shell design.

3. Locate \tilde{W}_{min} in $\bar{\alpha}_x - \bar{\alpha}_y$ space. There may be many points in the $\bar{\alpha}_x - \bar{\alpha}_y$ space with the same value of $\tilde{W} = \tilde{W}_{min}$. Find the upper bound on l_x (or l_y) by substituting the value of l_y (or l_x) from Step 2 by taking care that

$$\sigma_{xy_{sk_{cr}}} > \sigma_{xy_{sk}} .$$

4. If a MG design is desired then find the lower bounds on l_x and l_y from a consideration that t_x and t_y must be greater than or equal to the specified MG. If G = the specified MG then for different types of stiffeners this leads to

$$\text{RS;RR:} \quad l_i \geq \frac{G \bar{\alpha}_i (1-v^2)}{\bar{\lambda}_{ii}}$$

$$\begin{array}{l} \text{AS; AR, IAS;} \\ \text{IAR, TS; TR:} \end{array} \quad l_i \geq \frac{G \bar{\alpha}_i (1-v^2)}{\bar{\lambda}_{ii}} \left(\frac{1 + c_{fi} k_i}{1 + 4c_{fi} k_i} \right)^{1/2}$$

$$\begin{array}{l} \text{C, Z, IS;} \\ \text{C, Z, IR:} \end{array} \quad l_i \geq \frac{G \bar{\alpha}_i (1-v^2)}{\bar{\lambda}_{ii}} \frac{(1 + 2c_{fi} k_i)^{3/2}}{(1 + 6c_{fi} k_i)^{1/2}}$$

$$\text{HS; HR:} \quad l_i \geq \frac{G \bar{\alpha}_i (1-v^2)}{\bar{\lambda}_{ii}} \frac{(k_{1i} + 2 + 2k_{2i})^{1/2}}{(6k_{1i}^2 + 8k_{1i} + 16k_{2i} + 24k_{2i}^2 + 4)^{1/2}}$$

where i stands for x and y (stringer and ring parameters) in turn; there being no summation on i .

Obviously this step is redundant for a WMG design.

5. Within the bounds found in Steps 2 through 4 above, select values of l_x and l_y in order to give integer values for the number of stringers and rings. If the upper and the lower bounds on l_x and l_y from Steps 2 through 4 are not feasibly adoptable then move to another point having the same value of \tilde{W}_{\min} or to the next higher value as desired.

This will lead to a nonunique solution unless the number of rings or stringers is fixed at the minimum level.

6. Find $\sigma_{xy_{sk_{cr}}}$ with the adopted values of l_x and l_y and see that SB ($= \sigma_{xy_{sk}} / \sigma_{xy_{sk_{cr}}}$) is within the desired limit.

7. Find the panel buckling load, \bar{N}_{xy_p} and check for PB ($= \bar{N}_{xy} / \bar{N}_{xy_p}$). If panel buckling takes place, avoid it by adopting increased number of rings and/or stringers for the same point in $\bar{\alpha}_x - \bar{\alpha}_y$ space. If this is not possible then move to a different point in $\bar{\alpha}_x - \bar{\alpha}_y$ space having nearly same value of \tilde{W} or next higher value as desired.

8. Confirm that SB is not equal to PB. This avoids simultaneous occurrence of failure modes. If equal, separate them reasonably by adopting different values of l_x and/or l_y .

9. Calculate the depths of stiffeners, d_i from

$$\text{RS;RR:} \quad d_i = h \bar{\alpha}_i$$

$$\text{AS;AR, IAS;IAR, TS;TR:} \quad d_{wi} = h \bar{\alpha}_i \frac{(1 + c_{fi} k_i)}{(1 + 4c_{fi} k_i)^{1/2}}$$

$$\text{C,Z, IS;C,Z, IR:} \quad d_{wi} = h \bar{\alpha}_i \left(\frac{1 + 2c_{fi} k_i}{1 + 6c_{fi} k_i} \right)^{1/2}$$

$$\text{HS;HR:} \quad d_i = h \bar{\alpha}_i \frac{k_{1i} + 2 + 2k_{2i}}{(6k_{1i}^2 + 8k_{1i} + 16k_{2i} + 24k_{2i}^2 + 4)^{1/2}}$$

The value of d_i are generally physically realistic.

10. Find the thickness of the stiffeners from

$$\text{RS;RR:} \quad t_i = \frac{Eh \bar{\lambda}_{ii} \ell_i}{E_i d_i (1-\nu^2)}$$

$$\text{AS;AR,IAS;IAR,TS;TR:} \quad t_{wi} = \frac{E \bar{\lambda}_{ii} h \ell_i}{E_i d_{wi} (1-\nu^2) (1 + c_{fi} k_i)}$$

$$t_{fi} = c_{fi} t_{wi}; \quad b_{fi} = k_i d_{wi}$$

$$\text{C,Z,IS;C,Z,IR:} \quad t_{wi} = \frac{E \bar{\lambda}_{ii} h \ell_i}{E_i d_{wi} (1-\nu^2) (1 + 2c_{fi} k_i)}$$

$$t_{fi} = c_{fi} t_{wi}; \quad b_{fi} = k_i d_{wi}$$

$$\text{HS;HR:} \quad t_{wi} (= t_{li}) = \frac{E \bar{\lambda}_{ii} h \ell_i}{E_i d_{wi} (1-\nu^2) (k_{li} + 2 + 2k_{2i})}$$

$$t_{fi} = t_{2i} = c_{fi} t_{wi}; \quad b_{fi} (= b_{fx1}) = k_{li} d_x;$$

$$b_{fx2} = k_{2x} d_x$$

11. Calculate the weight of the shell from

$$W_{\text{shell}} = 2\pi R L h \rho_{sk} \bar{W}$$

12. Repeat the above steps for different $Z(h)$ values and plot W versus h . From the plot locate the value of $Z(h)$ for absolute minimum weight of the composite shell.

13. Finalize all the required dimensions and the weight of the

shell at the value of Z which yields the minimum weight design.

14. Repeat all the above steps for a different MG , if so desired.

Torsion Combined with Axial Compression
with or without Lateral Pressure

Once again it is required to solve determinant, Equation (9), for \bar{K}_s and minimize w.r.t. β to obtain $\bar{K}_{s_{cr}}$. But it was found that a fifth order determinant was no longer sufficient to give appreciably correct values of $\bar{K}_{s_{cr}}$ for all cases. At the same time with higher order of the determinant, the computer time increased rapidly. To obviate this trouble it is necessary to still consider a small order determinant and get the best possible accuracy for all cases. For this purpose, the determinant, Equation (9) is written in the following fashion

$$|[A] + \bar{K}_s[B]| = 0$$

If $\{X\}$ be the associated eigenvectors, then the results of Equation (8) written in matrix form is

$$[A] + \bar{K}_s[B] \{X\} = 0 \quad (30)$$

where $[A]$ is the diagonal matrix of γ_i ($i = 1, 2, \dots, \infty$) terms only; see Equation (10). The value of $\beta (= nL/\pi R)$ must be assumed in order to find the elements in \bar{A} . The value of β corresponding to the minimum possible value of n , i.e. 2 is substituted in order to find \bar{K}_s and then $\bar{K}_{s_{cr}}$ is found as explained later in

this section.

[B] is the matrix with all other terms except the diagonal terms are zero, with \bar{K}_s taken out as a common term.

It should be noted that \tilde{A} as well as \tilde{B} are symmetric matrices. Furthermore, \tilde{A} is positive definite (which is obviously true for the pure torsional loading, but it is also true for the combined loading if the shell does not buckle when $\bar{K}_s = 0$). Hence the minimum eigenvalue, $\bar{K}_{s_{\min}}$ is always real. Also, the sign of the eigenvalue does not have any effect on the solution. We find $\bar{K}_{s_{\min}}$, see Appendix D, from

$$\bar{K}_{s_{j+1}}^2 = [V_{2j+2}]^T \{V_{2j}\} / [V_{2j+2}]^T \{V_{2j+2}\} \quad (31)$$

where $j = 0, 1, 2, \dots$ are iteration numbers and \tilde{V} are calculated from

$$[A]\{V_{j+1}\} = -[B]\{V_j\} \quad (32)$$

The initial vector $\{V_0\}$ is arbitrarily chosen. The corresponding eigenvectors are given by

$$\begin{aligned} \{X_{j+1}^1\} &= \{V_{2j}\} + \bar{K}_{s_{j+1}} \{V_{2j+1}\} \quad \text{for positive } \bar{K}_s \\ \{X_{j+1}^2\} &= \{V_{2j}\} - \bar{K}_{s_{j+1}} \{V_{2j+1}\} \quad \text{for negative } \bar{K}_s \end{aligned} \quad (33)$$

The above iteration process is continued until the difference between the eigenvalues obtained from two successive iterations is less than a specified value, ϵ , for the convergence criterion to be satisfied.

Alternatively, from a knowledge of the eigenvectors the order of

the determinant can be reasonably reduced by considering only the dominant elements in an intermediate order determinant (with m and m' in Equation (9) not necessarily starting from 1, for example, it could be from 6 to 12). It is found that the order of the determinant which yields the absolute value of the lowest eigenvector not less than 10 to 15 percent of the highest value with the greatest eigenvector taking an intermediate position is sufficient to yield appreciably correct eigenvalues \bar{K}_s . To start with, one does not have any idea as to what intermediate order determinant suffices. However, by printing out all the elements of the eigenvectors for any arbitrarily selected initial range and order of determinant, one gets a clear picture as to which order will suffice. Provision is made in the program to adopt a higher or lower order determinant depending upon the absolute values of lowest element becoming more or less than the specified limit. Hence the following steps are adopted.

1. Choose the range of m and m' from apriori knowledge, if possible, otherwise arbitrarily. Excepting for the computer time, this choice does not affect the final result. To start with m and m' are taken arbitrarily equal to one and 15 respectively.

2. Compute \bar{K}_s , \tilde{X}_1^1 and \tilde{X}_2^2 for the above range of m and m' from Equations (31), (32) and (33). The criterion for convergence adopted in this work is that $(\bar{K}_{s_1} - \bar{K}_{s_2})/\bar{K}_{s_2} \leq 0.01$ where \bar{K}_{s_1} and \bar{K}_{s_2} are the values of \bar{K}_s at subsequent iterations.

3. Check the elements of the eigenvectors \tilde{X}_1^1 and \tilde{X}_2^2 and adopt a new set of values of m and m' , if required, to include only those elements which influence the buckling mode the most. The criterion

adopted in this work is that for randomly selected parameters the value of \bar{K}_s obtained with the lowest order determinant should not differ from a reasonably higher order determinant by more than .015 percent.

4. Finally, calculate \bar{K}_s , \bar{X}^1 and \bar{X}^2 from Equations (31), (32) and (33). Provision is made in the program to adopt a new set of m and m' values if the lowest values in the elements of \bar{X}^1 and \bar{X}^2 become more than 15 percent of the most influencing amplitude.

Having found the lowest eigenvalue as explained above, $\bar{K}_{s_{cr}}$ is once again obtained by minimizing \bar{K}_s w.r.t. β . Instead of treating β as a continuous variable and applying the golden section method, it is found more economical (computer timewise) to apply a sequential search technique with varying step sizes by evaluating \bar{K}_s at values of β corresponding to numerical values of $n(\geq 2)$ and comparing the subsequent values. An approximate formulation (not employed herein) is presented in Appendix E for estimating $\bar{K}_{s_{cr}}$ for a given geometry and value of the applied load.

Next, it is necessary to perform an unconstrained minimization of \bar{W}^* , see Equation (24), w.r.t. the independent variables $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$ for fixed Z and at different values of $\bar{\alpha}_x$ and $\bar{\alpha}_y$ as in the pure torsional loading case. An important and distinct feature of the combined load case is that for arbitrarily selected starting $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$ at fixed $\bar{\alpha}_x$, $\bar{\alpha}_y$ and Z a no-solution zone is encountered because the diagonal matrix A in Equation (30) is no longer positive definite. This occurs when one or more of the γ_m in Equation (10) have nonpositive value. Physically this means that the shell will buckle under the applied axial load without application of the torsional load. The only way to

circumvent this difficulty is to find the optimizing $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$, at each point in $\bar{\alpha}_x - \bar{\alpha}_y$ space for fixed Z where a solution is desired, required to withstand only the axial compression; increase $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$ so that buckling without torsional load does not take place; and then solve for the combined load case. Since the aim of this work is in no way to develop a solution for axial compressive load, advantage is taken of the work done by previous investigators. By extending the work of Reference [31] from panel to a complete cylinder the solution to the axial compressive load is found.

In order to minimize \bar{W} with the constraint $\bar{K}_{s_{cr}}^* = Z\bar{N}_{xy}^*$, the augmented objective function is reformulated from Equation (24) as follows (see Reference [37])

$$\bar{W}^* = \bar{W} + \lim_{\lambda^* \rightarrow \infty} \lambda^* (\bar{K}_{s_{cr}}^* - Z\bar{N}_{xy}^*)^2 \quad (34)$$

It should be noted that for the case of torsional loading λ^* was assigned a high (10^6) value and the same value was maintained during minimization. Also the bracketed term was not squared. For the combined load cases λ^* is increased gradually during optimization procedure from a smaller value to 10^{15} and the bracketed term is squared. This way during the initial part of iterations when the term in the bracket is greater than unity then λ^* has a smaller value and when it becomes less than unity then λ^* attains a higher value. This way the constraint is applied gradually. This helps in reducing computer time. Once again an unconstrained minimization of \bar{W}^* is performed w.r.t. $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$ in $\bar{\alpha}_x - \bar{\alpha}_y$ space at fixed Z by the flexible polyhedron type of simplex method

referred earlier in this chapter. In order to make \bar{W}_{\min}^* independent of the starting $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$ as far as possible, the best choice of initial λ^* is such that the initial value of $\lambda^*(\bar{K}_s^* - Z\bar{N}_{xy}^*)^2$ is of the same order as \bar{W} .

Fixing the range of Z-values becomes quite simple from a knowledge of the solutions of pure torsion and axial compression loads separately. From a practical consideration and for thin ring theory to hold ($R/d_{wy} \geq 20$), the upper bounds on $\bar{\alpha}_x$ and $\bar{\alpha}_y$ are fixed. In essence, the procedure for minimum weight design has to be to select different Z-values, generate data for different Z and within the bounds of $\bar{\alpha}_x$ and $\bar{\alpha}_y$ as mentioned above, finalize the design with the types of stiffeners selected by satisfying all other constraints, change stiffeners' shapes and sizes to see their effect on the minimum weight, plot W versus h and obtain the final design. The constraints to be satisfied are (i) geometric constraint on the dimensions for a minimum gauge design, if required, (ii) stress levels in the skin and stiffeners obtained from Equations (17) must be such that yielding of the materials does not take place (von Mises-Hencky yield criterion, see Equation (20), is used for the skin), (iii) the critical stresses in the skin and stringer must be greater than the actual stresses (the ring being in tension does not buckle), (iv) panel instability does not take place, for which \bar{N}_{xy_p} should be greater than \bar{N}_{xy} and PB ($=\bar{N}_{xy}/\bar{N}_{xy_p}$) is within allowable limits and lastly, (v) simultaneous occurrence of failure modes is avoided.

The final design is achieved in five distinct stages including two separate computer programs called 'OPTIMUM' and 'CHECK'. In Stage 1 the range of Z-values, upper bounds of $\bar{\alpha}_x$ and $\bar{\alpha}_y$, and the minimum number

of rings are decided upon. Then data are generated in Stage 2 from program 'OPTIMUM' at a fixed Z-value. This program first finds the minimum of $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$ to withstand the compressive load alone to facilitate avoiding a no-solution zone. Next for point in the $\bar{\alpha}_x - \bar{\alpha}_y$ space it optimizes \bar{W}^* , Equation (34), w.r.t. $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$ to withstand the combined loads. Also the upper and lower bounds on the spacings of the stiffeners for the satisfaction of some of the constraints and all the stresses in the skin and stiffeners are found from this program. This marks the end of Stage 2. The number of stringers and rings are fixed within the bounds of l_x and l_y found from program 'OPTIMUM' in Stage 3. The data, found above, are then used in Stage 4 and fed in program 'CHECK' to obtain a minimum weight design by ensuring that none of the constraints is violated. Golden section method is used for seeking \bar{N}_{xy_p} by minimizing \bar{K}_s w.r.t. β in program 'CHECK' as it is found to be more economical than using discrete values of β because of large number of associated circumferential waves. In the final Stage 5 all the dimensions and weight of the shell are finalized.

Arranging the solution procedure as outlined above provides great savings of computer time. The steps to be followed in obtaining a final minimum weight design are listed below.

Design Steps for Combined Load Cases:

For nomenclatures of the stiffeners, one should see the Glossary of Abbreviations given in the beginning. Dimensional notations are shown in Figure B1. It should be noted that for any given problem the known quantities are (1) applied loads, \bar{N}_{xx} , q and \bar{N}_{xy} ; (2) the radius and length of the shell; (3) the skin and stiffener materials and their

properties; and (4) the position of the stiffeners (inside in the present work). As some of the steps are common for all types of stiffeners, it is considered wise to list them together. For convenience the steps have been categorized under the five distinct stages explained earlier. If a step is same for all types of stiffeners, no mention is made of the type of stiffening.

Stage 1. Preliminary Decisions

The following steps are easily implemented without the help of computer (a calculator may be sufficient) and are required before starting the optimization procedure.

1. Determine the range of Z-values from a prior knowledge of the design results of pure torsional load and compressive load. Find h for each value of Z from Equation (5). Complete design for the lowest Z-value first, then for the next higher Z-value and so on by following the steps mentioned below.

2. Find the range of $\bar{\alpha}_x$ from practical considerations and of $\bar{\alpha}_y$ from a consideration of thin ring theory (as the stress in the rings is based on thin ring theory) which requires that R/d_{wy} must be greater than or equal to 20. Hence the following:

$$\text{RR} : \bar{\alpha}_y \leq \frac{R}{20h} ; \text{AR, IAR, TR: } \bar{\alpha}_y \leq \frac{R}{20h} \frac{(1 + 4c_{fy}k_y)^{1/2}}{1 + c_{fy}k_y}$$

$$\text{CR, ZR, IR: } \bar{\alpha}_y \leq \frac{R}{20h} \left(\frac{1 + 6c_{fy}k_y}{1 + 2c_{fy}k_y} \right)^{1/2}$$

$$\text{HR} : \bar{\alpha}_y \leq \frac{R}{20h} \frac{(6k_{1y}^2 + 8k_{1y} + 16k_{2y} + 24k_{2y}^2 + 4)^{1/2}}{k_{1y} + 2 + 2k_{2y}}$$

3. Decide about the minimum number of rings to be used for the smeared technique to hold true. Three is taken here [41]. This step will give an upper bound on l_y .

Stage 2. Optimization (Program 'OPTIMUM')

The following steps are built-in in program 'OPTIMUM'. They are given here with their expressions, wherever possible, to make them explicit for the benefit of the designer who wants to understand the details of the solution procedure. Subscript i stands for x and y , indicating stringer and ring parameters in turn (no summation on i).

1. Within the bounds decided upon in step 2 of Stage 1 and at certain intervals in $\bar{\alpha}_x - \bar{\alpha}_y$ space, generate data by first solving for simple compressive load and then for the combined load, viz., find $\bar{\lambda}_{xx}$, $\bar{\lambda}_{yy}$, \bar{W} , \bar{K}_s , β , n and any other value to check for convergence by using the optimization procedures discussed earlier in this chapter. To start with, it is profitable to take a bigger interval and first select points at one fixed $\bar{\alpha}_x$ (say ten) and at varying $\bar{\alpha}_y$. By seeing stress levels in the skin and stiffeners, mentioned in the following step, one can avoid generating data in the complete region where yielding of materials takes place.

2. Calculate stresses in the skin and stiffeners by employing Equation (17) and find stress σ_s in the skin by Equation (20) for checking the yielding of the skin material.

3. Compute the stringer and ring depths, d_i from the definitions

of $\bar{\alpha}_i$.

$$\text{RS;RR:} \quad d_i = h \bar{\alpha}_i$$

$$\text{AS;AR, IAS; IAR, TS;TR:} \quad d_{wi} = h \bar{\alpha}_i \frac{(1 + c_{fi} k_i)}{(1 + 4c_{fi} k_i)^{1/2}}$$

$$\text{C,Z, IS;C,Z, IR:} \quad d_{wi} = h \bar{\alpha}_i \left(\frac{1 + 2c_{fi} k_i}{1 + 6c_{fi} k_i} \right)^{1/2}$$

$$\text{HS;HR:} \quad d_i = h \bar{\alpha}_i \frac{k_{1i} + 2 + 2k_{2i}}{(6k_{1i}^2 + 8k_{1i} + 16k_{2i} + 24k_{2i}^2 + 4)^{1/2}}$$

4. Find an upper bound on stringer spacing, l_x from the fact that $|\sigma_{xx_{sk_{cr}}}| > |\sigma_{xx_{sk}}|$ by neglecting the small values of $\sigma_{xy_{sk}}$ and $\sigma_{yy_{sk}}$ and adopting a stringer stress-spacing factor, k_{lx} to account for them. For torsion combined with axial compression a good estimate for k_{lx} is found to be about 0.95. With lateral pressure included this factor can be taken more than one. From Bulson [30]

$$\sigma_{xx_{sk_{cr}}} = \frac{\pi E}{3(1-\nu^2)} \left(\frac{h}{l_x} \right)^2$$

so for $|\sigma_{xx_{sk_{cr}}}| > |\sigma_{xx_{sk}}|$ and k_{lx} included, we have

$$l_x < h k_{lx} \left(\frac{\pi E}{3(1-\nu^2) |\sigma_{xx_{sk}}|} \right)^{1/2}$$

Note that for a HS this expression gives the effective stringer spacing, l_{ex} and $l_x = l_{ex} + b_x$ where $b_x = k_{ix} d_x$ (see Figures 1 and B1).

5. If a specified minimum gauge, MG ($=G$, as for example G could be .05) is desired on all dimensions then from a consideration of the stringer and ring thicknesses t_{wi} to be greater than or equal to G , the lower limits on l_x and l_y are found as follows:

$$\text{RS;RR:} \quad l_i \geq \frac{G \bar{\alpha}_i (1-v^2)}{\bar{\lambda}_{ii}}$$

$$\text{AS;AR, IAS; IAR, TS; TR:} \quad l_i \geq \frac{G \bar{\alpha}_i (1-v^2)}{\bar{\lambda}_{ii}} \frac{(1 + c_{fi} k_i)^{1/2}}{(1 + 4c_{fi} k_i)^{1/2}}$$

$$\text{C, Z, IS; C, Z, IR:} \quad l_i \geq \frac{G \bar{\alpha}_i (1-v^2)}{\bar{\lambda}_{ii}} \frac{(1 + 2c_{fi} k_i)^{3/2}}{(1 + 6c_{fi} k_i)^{1/2}}$$

$$\text{HS;HR:} \quad l_i \geq \frac{G \bar{\alpha}_i (1-v^2)}{\bar{\lambda}_{ii}} \frac{(k_{1i} + 2 + 2k_{2i})^2}{(6k_{1i}^2 + 8k_{1i} + 16k_{2i}^2 + 24k_{2i} + 4)^{1/2}}$$

This step is not required for a design without minimum gauge (WMG).

6. Substituting the values of d_x from step 3 and l_x from step 4 find the thickness of the stringer, t_{wx} and t_{fx} from the definitions of $\bar{\lambda}_{xx}$. Also find the stringer flange width, b_{fx} .

$$\text{RS:} \quad t_x = \frac{Eh\bar{\lambda}_{xx}l_x}{E_x d_x (1-v^2)}$$

$$\text{AS, IAS, TS: } t_{wx} = \frac{E \bar{\lambda}_{xx} h \ell_x}{E_x d_{wx} (1 - \nu^2) (1 + c_{fx} k_x)}$$

$$t_{fx} = c_{fx} t_{wx}; \quad b_{fx} = k_x d_{wx}$$

$$\text{C, Z, IS: } t_{wx} = \frac{E \bar{\lambda}_{xx} h \ell_x}{E_x d_{wx} (1 - \nu^2) (1 + 2c_{fx} k_x)}$$

$$t_{fx} = c_{fx} t_{wx}; \quad b_{fx} = k_x d_{wx}$$

$$\text{HS: } t_{wx} (= t_{lx}) = \frac{E \bar{\lambda}_{xx} h \ell_x}{E_x d_{wx} (1 - \nu^2) (k_{lx} + 2 + 2k_{2x})}$$

$$t_{fx} = t_{2x} = c_{fx} t_{wx}; \quad b_{fx} (= b_{fx1}) = k_{lx} d_x; \quad b_{fx2} = k_{2x} d_x$$

7. Find the upper bound on the ring spacing, ℓ_y from the condition that $|\sigma_{xx_{stw_{cr}}}|$ or $|\sigma_{xx_{stf_{cr}}}| > |\sigma_{xx_{st}}|$ (see expressions for $\sigma_{xx_{stw_{cr}}}$ in Table 1). A ring stress-spacing factor, k_{ly} is adopted to help get an upper limit on ℓ_y so as to obtain an acceptable stringer flange (or web) buckling coefficient, STFB (or STWB) finally. The values of ℓ_x from step 4 and of t_{wx} , t_{fx} and b_{fx} from step 6 are substituted in the equations below. Hence the following:

$$\text{RS: } \ell_y < k_{ly} m \ell_x \bar{\alpha}_x \bar{\lambda}_{xx} \left(\frac{E_x}{12 h \bar{\alpha}_x^2 (1 - \nu^2)^3 |\sigma_{xx_{st}}| - .407 \pi^2 E_x \ell_x^2 \bar{\lambda}_{xx}} \right)^{1/2}$$

$$\text{AS: } \ell_y < k_{\ell_y} \frac{d_{wx}^2}{\left[\frac{12(1-\nu^2)}{\pi E_x} \left(\frac{d_{wx} - t_{fx}}{t_{wx}} \right)^2 |\sigma_{xx_{st}}| - .407 \right]^{1/2}}$$

$$\text{IAS, TS: } \ell_y < k_{\ell_y} \frac{d_{fx}^2}{\left[\frac{12(1-\nu^2)}{\pi E_x} \left(\frac{d_{fx}}{t_{fx}} \right)^2 |\sigma_{xx_{st}}| - .407 \right]^{1/2}}$$

where for IAS: $d_{fx} = b_{fx} - t_{wx}$ and for TS: $d_{fx} = (b_{fx} - t_{wx})/2$

$$\text{C,Z,IS: } \ell_y < k_{\ell_y} \frac{b_{fx}/2}{\left[\frac{12(1-\nu^2)}{\pi E_x} \left(\frac{b_{fx}}{2t_{fx}} \right)^2 |\sigma_{xx_{st}}| - .407 \right]^{1/2}}$$

The expressions for $\sigma_{xx_{stw_{cr}}}$ and $\sigma_{xx_{stf_{cr}}}$ for HS do not contain ℓ_y and hence no bound on ℓ_y from this requirement is obtained with HS.

Note that in the above equations if the expression in the denominator whose square root is desired becomes negative, which usually happens in the present work, then any value of ℓ_y will satisfy the requirement $|\sigma_{xx_{stw_{cr}}}|$ or $|\sigma_{xx_{stf_{cr}}}| > |\sigma_{xx_{st}}|$, what is desired.

This marks the end of Stage 2 and of all operations required in program 'OPTIMUM'.

Stage 3: Feasibility Decisions

1. With the data generated from stage 2, locate the feasible design space by discarding all the unfeasible points, which we call unfeasible design space distinguished by the following characteristics:

(i) the points where yielding of skin and/or stiffener materials takes place are in unfeasible design space;

(ii) the points where the conditions in steps 4 and 5 of Stage 2 for ℓ_x as well as the conditions in step 3 of Stage 1 and steps 5 and 7 of Stage 2 for ℓ_y are simultaneously not satisfied fall in unfeasible design space.

2. Taking care of the limitations in step 3 of Stage 1 and steps 4, 5 and 7 of Stage 2 adopt suitable ℓ_x and ℓ_y to give numerical values of the number of stringers and rings by first starting with the point giving lowest \bar{W} in the feasible region and then moving to the other point having same value of \bar{W} , if possible, or to the next higher value of \bar{W} if any of the constraints mentioned below and built-in in program 'CHECK' are violated. Note that the stiffener spacings ℓ_x and ℓ_y can always be adjusted within the limits found from program 'OPTIMUM' of Stage 2.

Stage 4: Checking of Constraints (Program 'CHECK')

1. For the adopted values of ℓ_y and the generated data, viz., $\bar{\alpha}_x$, $\bar{\lambda}_{xx}$ and associated values of \bar{e}_x , $\bar{\rho}_{xx}$ find the panel instability load, \bar{N}_{xy_p} and number of circumferential waves, n_p for panel buckling and check for the panel buckling coefficient, PB.

2. For the adopted values of ℓ_x and ℓ_y and the generated data, $\sigma_{xx_{sk}}$, $\sigma_{yy_{sk}}$, and $\sigma_{xy_{sk}}$, find the skin buckling stress $\sigma_{xy_{sk_c}}$ and check for the skin buckling coefficient, SB.

3. For the adopted value of ℓ_y , d_{wx} , b_{fx} , t_{wx} , t_{fx} find the critical stresses in the stringer web and flange, $\sigma_{xx_{stw_{cr}}}$ and $\sigma_{xx_{stf_{cr}}}$

with the help of equations in Table 1 and check for the stringer web and flange buckling coefficients, STWB and STFB respectively.

4. Check for the simultaneous occurrence of failure modes and avoid it.

5. If any of the failure modes, mentioned in steps 1 through 4 are violated, first try to avoid it by adjusting l_x and l_y , within the limits found in Stages 1 and 2 and feeding the new data in program 'CHECK'. If this is not possible try to satisfy all the constraints by moving to another point with same value of \bar{W} . If this too is not possible move to a point having higher \bar{W} and repeat steps 1 through 4 once again.

The lowest value of \bar{W} where all the constraints are satisfied is the feasible design point. This marks the end of stage 4 and the operations in program 'CHECK'.

Stage 5: Finalization

1. Finalize all the dimension of the rings, i.e., find d_{wy} (or d_y), t_{wy} (or t_y), t_{fy} , b_{fy} from the definitions of $\bar{\alpha}_y$ and $\bar{\lambda}_{yy}$. One needs simply to change the subscript x to y in steps 3 and 6 of Stage 2 to get expressions for the ring parameters in place of those of the stringer.

2. Find the weight of the composite shell from

$$W = 2\pi RLh \rho_{sk} \bar{W}$$

3. Repeat all the above steps for different Z-values (at least three are needed) if minimum weight design with any pre-decided

combination of stiffener shapes and sizes are desired, or for same Z-value but with different shapes and sizes of stiffeners when a comparative study of the effects of stiffeners' geometries on the cylinder weight for fixed Z is sought for.

4. Plot W versus h and locate the value of h (and hence Z) giving W_{\min} for the type of stiffeners that minimum weight design is required. Repeat all the above steps for the Z-value giving W_{\min} if exact minimum weight configurations are required.

5. If the effect of varying MG on the weight of the cylinder is desired then repeat all the above steps once again with the new MG.

For any combination of stringer-ring geometries, one has to simply pick up the constraint corresponding to the geometry selected from each of the steps listed above.

An example solution is given in Appendix F and the computer program developed is given in Appendix G.

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CHAPTER IV

EXAMPLES, NUMERICAL RESULTS AND DISCUSSION

Two design examples for the pure torsional loading case and one each for the combined loading cases (with and without lateral pressure) are presented. For all the four examples, the material properties are: $\nu = 0.33$, $\rho_{sk} = \rho_x = \rho_y = 0.101 \text{ lb/in}^3$, $E = E_x = E_y = 10.5 \times 10^6$ psi, $\sigma_{xy_{sk_y}} = 26,000$ psi, $\sigma_o = 45,000$ psi.

Example 1:

This example represents a pure torsional loading case with a typical C-141 fuselage. Load \bar{N}_{xy} is calculated from data provided by the Lockheed-Georgia Company, and representative of a critical flight maneuver that induces torsion in the fuselage.

$$R = 85.0 \text{ in.}, \quad L = 100.0 \text{ in.}$$

$$\bar{N}_{xy} = 418.538 \text{ lb/in.}, \quad \bar{N}_{xy}^* = 3.153 \times 10^{-7}$$

Example 2:

This example also represents a case of pure torsion and the geometry corresponds to the one used in Reference [16]. The value for \bar{N}_{xy} is arbitrary.

$$R = 95.5 \text{ in.}, \quad L = 291.0 \text{ in.}$$

$$\bar{N}_{xy} = 125.0 \text{ lb/in.}, \quad \bar{N}_{xy}^* = 1.8623 \times 10^{-9}$$

Example 3:

This example represents a loading case of torsion combined with axial compression (without lateral pressure) with C-141 fuselage. The axial compression, \bar{N} , value corresponds to the maximum bending stress resultant for the same flight maneuver mentioned in Example 1.

$$\begin{aligned} R &= 85.0 \text{ in.} , & L &= 100.0 \text{ in.} \\ \bar{N} &= 2700.0 \text{ lb/in.}, & \bar{N}_{xy} &= 418.538 \text{ lb/in.} \end{aligned}$$

Example 4:

This example represents the loading case of torsion combined with axial compression and internal lateral pressure with C-141 fuselage.

$$\begin{aligned} R &= 85.0 \text{ in.} , & L &= 100.0 \text{ in.} \\ \bar{N} &= 2700.0 \text{ lb/in.} , & \bar{N}_{xy} &= 418.538 \text{ lb/in.} \\ q &= 0.46277 \frac{\bar{N}}{R} (= 14.7 \text{ psi}) \end{aligned}$$

Internal stiffening has been used in all examples. The design results and the discussion of results are taken up separately for each one of the three loading cases.

Pure Torsional Load

The results of final minimum weight design for Example 1 are given in Tables 2 and 3 and for Example 2 in Tables 4 and 5. Figures 4 to 8 show the variations of shell weight, W , w.r.t. the skin thickness, h , for a WMG (without minimum gauge) design and for a varying MG (minimum gauge) design with different types of stiffeners for Example 1. All possible combinations of torsionally weak open-section types of stiffeners, for which weight savings could be anticipated, have been

Table 2. Final Minimum Weight Design Results
for Example 1 with Various Stiffeners.
($l_x, l_y \leq 25$ inches, Minimum 3 Rings)

	Type of Stiffening						
	RS-RR WMG	RS-RR MG = .05	TS-RR MG = .05	TS-TR MG = .05	IS-RR MG = .05	HS-RR MG = .05	HS-HR MG = .05
W	94.70136	475.152996	402.744165	417.987270	416.097539	406.335744	404.236345
h	.015865	.050480	.050480	.050480	.050480	.050480	.050480
t_x	.00003	.050191	.050314	.050384	.051213	.064204	.052132
t_y	.00200	.050117	.055709	.050616	.050999	.050489	.050818
d_x	14.59120	2.12016	5.24592	7.694040	3.747870	2.197086	2.35395
d_y	8.56440	4.745120	.504800	.480877	.631000	.075720	.062772
l_x	.593410	3.787744	24.276000	19.074000	21.362880	24.276000	24.276000
l_y	25.00000	25.00000	3.571428	4.000000	3.703703	1.250000	2.173913
GB	1.0	1.0	1.0	1.0	1.0	1.0	1.0
PB	.235610	.142331	.000751	.000787	.001538	.000085	.000257
SB	.714690	.886766	.789136	.973860	.843443	.098041	.29535
n	4	2	11	14	9	7	7
n_p	215	76	869	824	673	980	570

Table 3. Final Minimum Weight Design Results
for Example 1 with Gauge Variation and
Unbounded l_x, l_y with TS-RR, HS-RR and HS-HR.

	MG = .04 with $l_x, l_y \leq 25$ in.			MG = .05 with l_x, l_y unbounded		
	TS-RR	HS-RR	HS-HR	TS-RR	HS-RR	HS-HR
W	339.423055	334.956790	344.523878	315.990806	304.083322	327.922797
h	.041132	.041132	.041132	.050480	.05048	.05048
t_x	.040255	.040759	.041971	.053949	.071801	.055087
t_y	.040499	.040208	.040696	.051362	.050448	.051484
d_x	6.500832	2.557464	2.531826	13.114800	4.708042	4.08018
d_y	.308490	.123396	.076722	.126200	.07572	.062772
l_x	24.276000	24.276000	24.276000	178.02400	267.036	133.518
l_y	2.222222	1.020408	1.851851	3.030303	3.333333	3.030303
GB	1.0	1.0	1.0	1.0	1.0	1.0
PB	.000247	.000057	.000189	.000290	.000591	.000716
SB	.570336	.120849	.396824	.577205	.698488	.577108
n	16	7	7	24	7	7
n_p	1811	1236	679	1267	395	432

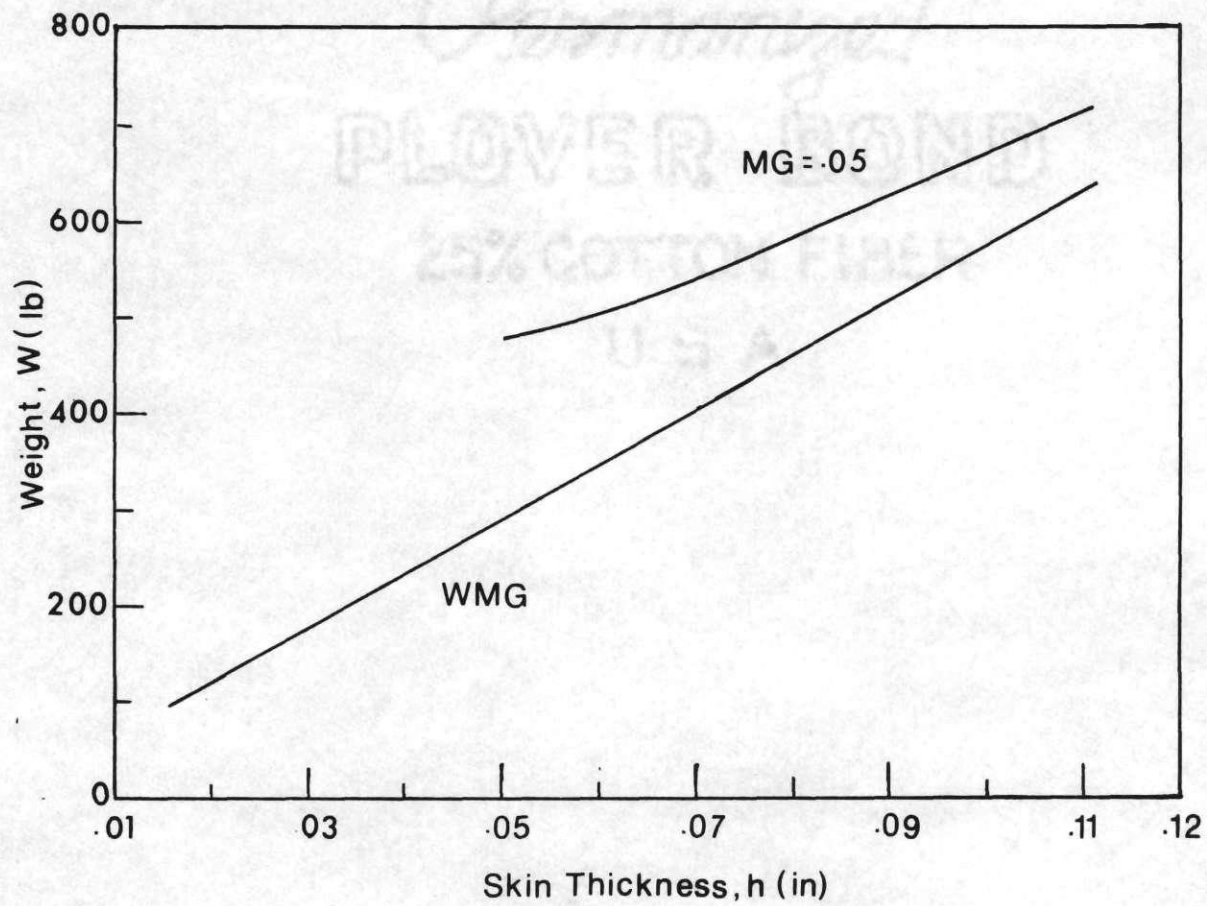


Figure 4: Minimum Weight Design for Example 1 with RS-RR.

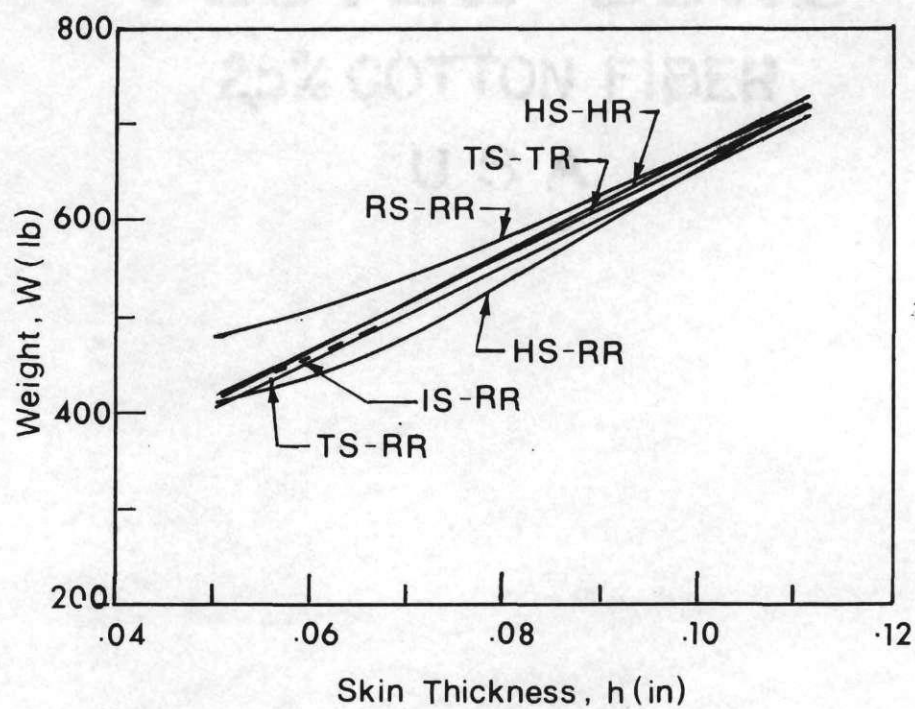


Figure 5: Minimum Weight Design for Example 1
With Various Types of Stiffeners.
($MG = .05$; $l_x, l_y \leq 25$)

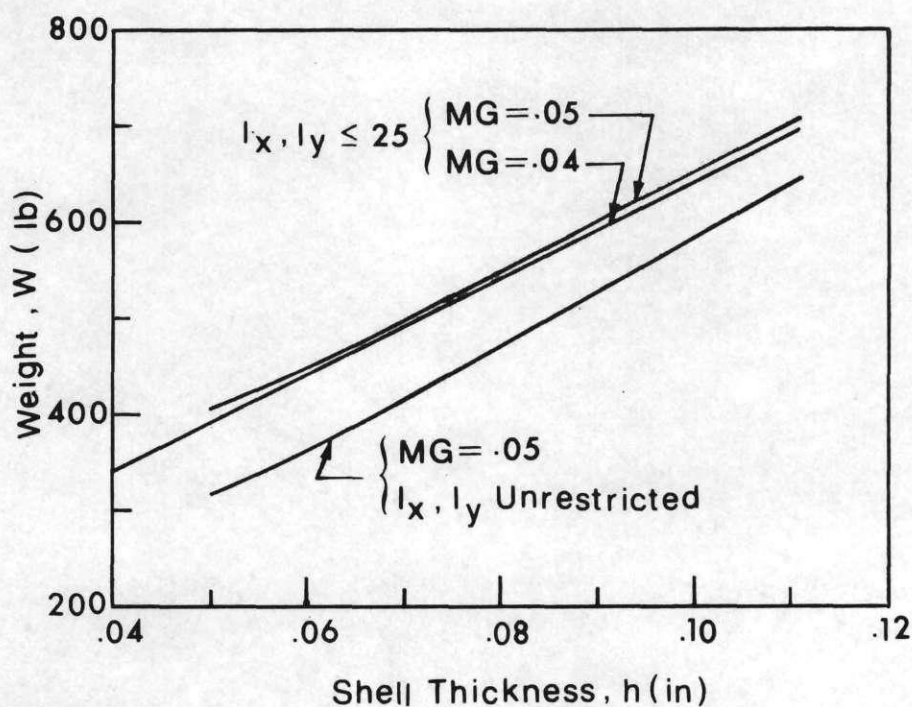


Figure 6: Effect of Gauge Variation and Relaxing Maximum l_x, l_y Limits on the Minimum Weight Design of Example 1 with TS-RR.

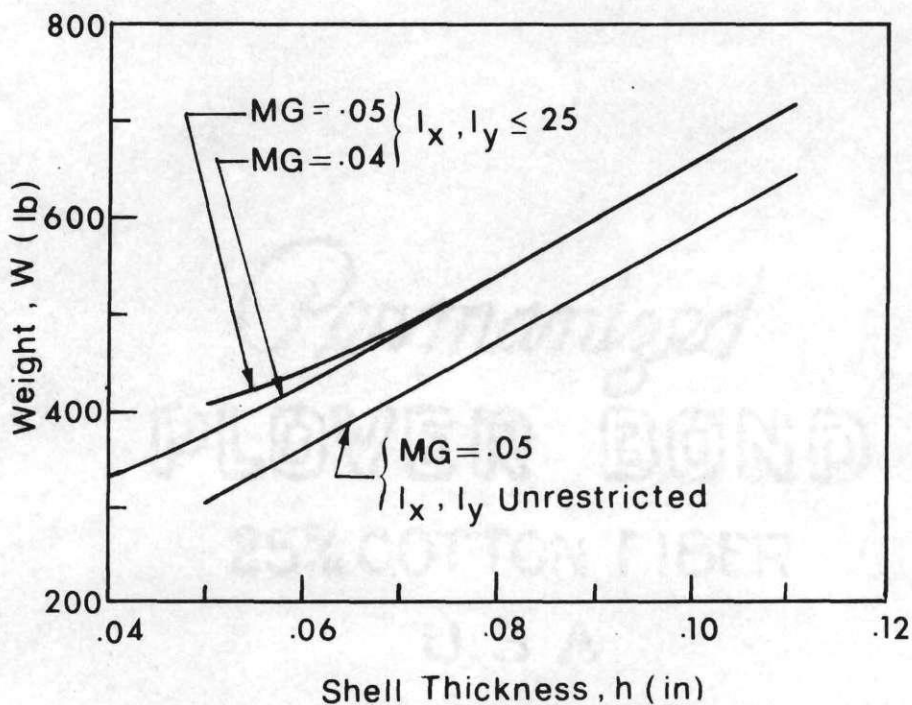


Figure 7: Effect of Gauge Variation and Relaxing Maximum l_x, l_y Limits on the Minimum Weight Design of Example 1 with HS-RR.

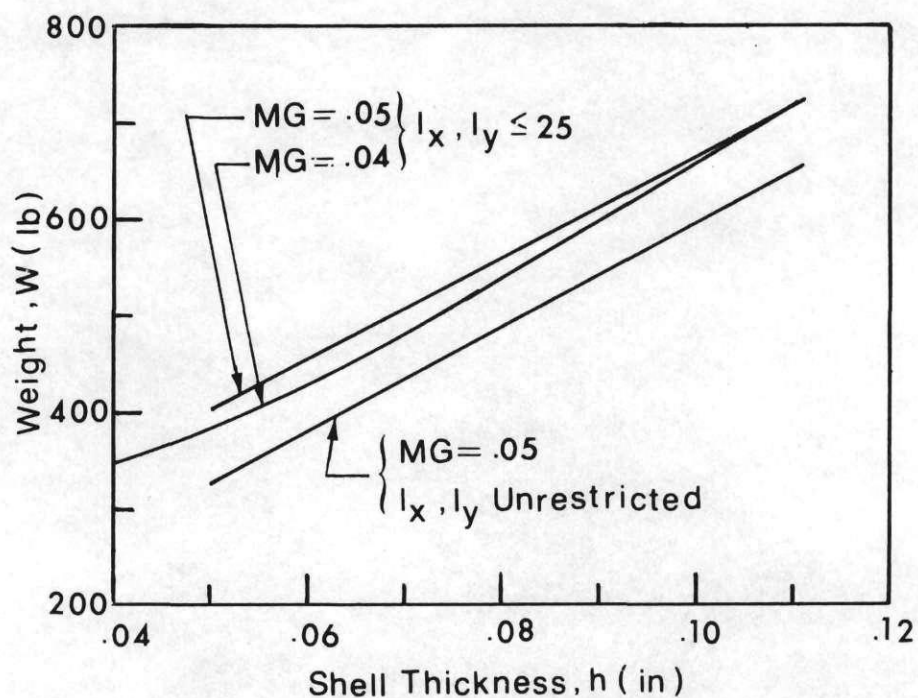


Figure 8: Effect of Gauge Variation and Relaxing Maximum l_x, l_y Limits on the Minimum Weight Design of Example 1 with HS-HR.

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Table 4. Final Minimum Weight Design Results
for Example 2 with Various Stiffeners.
($l_x, l_y \leq$ about 25 inches)

	Type of Stiffening				
	RS-RR WMG	RS-RR MG = .02	TS-RR MG = .02	HS-RR MG = .02	HS-HR MG = .02
W	93.312526	726.282488	717.688387	677.907820	788.824640
h	.004923	.020415	.020415	.020415	.020415
t_x	.000002	.03349	.020293	.020157	.020420
t_y	.000025	.020841	.020013	.020078	.027076
d_x	44.307000	.408300	10.960980	4.44255	3.807900
d_y	39.384000	9.390900	.510375	.408300	.095197
l_x	.218198	1.869300	23.078676	25.0019	15.790673
l_y	22.384615	14.550000	1.754519	1.785276	1.653409
GB	1.0	1.0	1.0	1.0	1.0
PB	.001357	.918003	.000039	.000021	.000018
SB	.963748	.979852	.870123	.90137	.769746
n	2	2	6	3	3
n_p	14	88	5535	761	778

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Table 5. Final Minimum Weight Design Results for Example 2 with Gauge Variation and Unbounded l_x, l_y with TS-RR and HS-RR.

	MG = .03 with $l_x, l_y \leq$ about 25 in.		MG = .02 with l_x, l_y Unbounded	
	TS-RR	HS-RR	TS-RR	HS-RR
W	944.859354	930.436201	643.218186	585.943385
h	.030001	.030001	.020415	.020415
t_x	.030584	.030061	.021019	.020474
t_y	.030031	.030211	.020057	.020244
d_x	9.353160	3.823865	13.789620	5.077200
d_y	.600020	.525017	.408300	.204150
l_x	24.001824	24.001824	37.502850	37.50285
l_y	3.163043	3.306818	1.835001	1.711764
GB	1.0	1.0	1.0	1.0
PB	.000018	.000074	.000035	.000019
SB	.883437	.964423	.954203	.830519
n	6	3	8	3
n_p	525	416	5622	820

Optimizing shape factors for the nonrectangular stiffeners in Tables 2, 3, 4, and 5 are as follows:

TS, IS and TR: $C_{fx} = 1.0 = C_{fy}$ and $k_x = 0.5 = k_y$.
 $(C_x = C_y = 1.155)$

HS and HR: Uniform thickness and $k_{1x} = 1.75 = k_{1y}$;
 $k_{2x} = 0 = k_{2y}$.

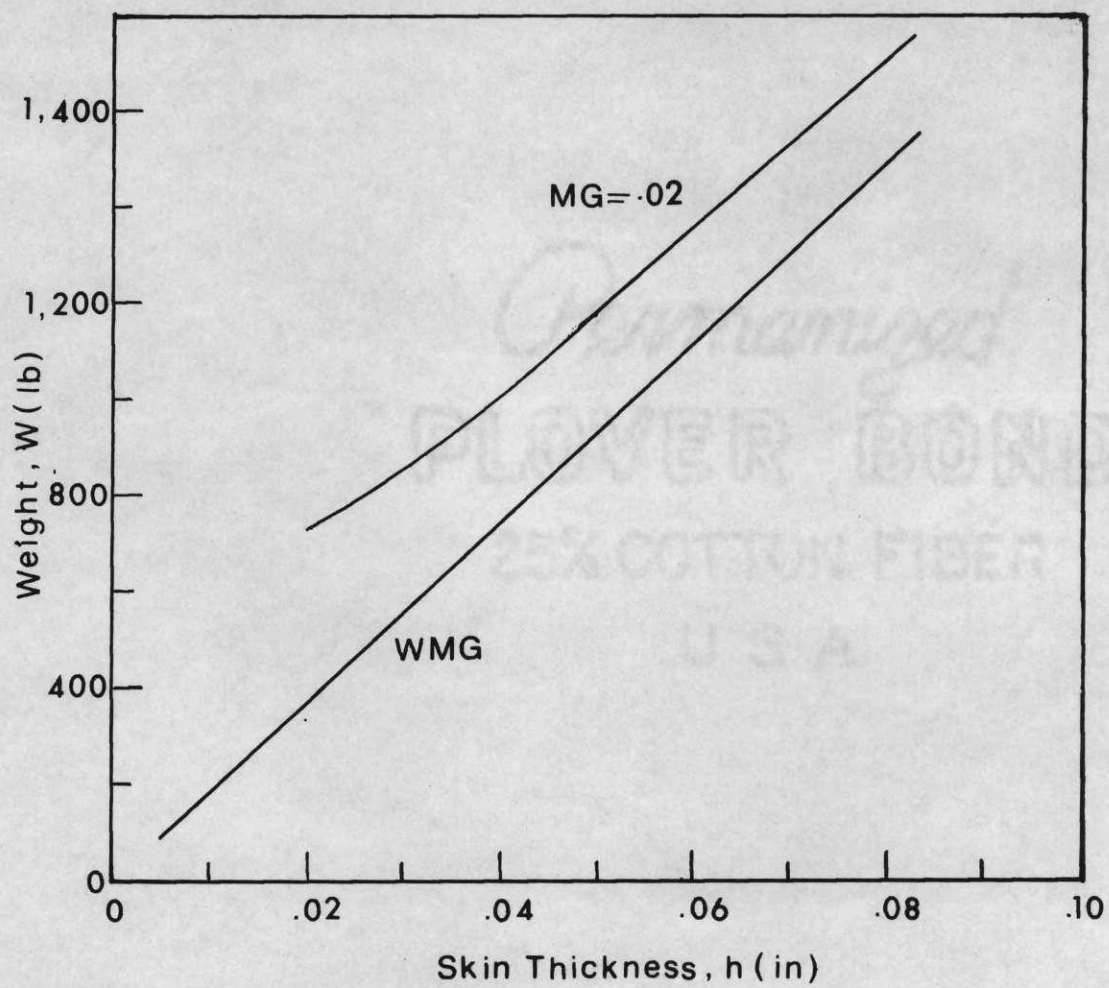


Figure 9: Minimum Weight Design of Example 2 with RS-RR.

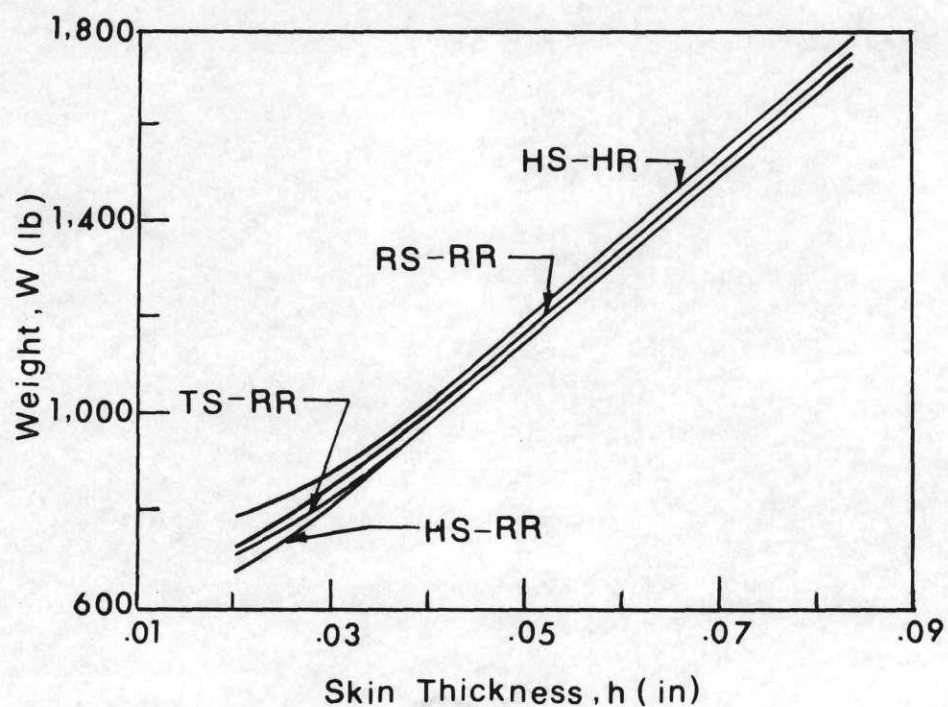


Figure 10: Minimum Weight Design of Example 2 with Various Types of Stiffeners. ($MG = .02$; $l_x, l_y \leq \text{about } 25$)

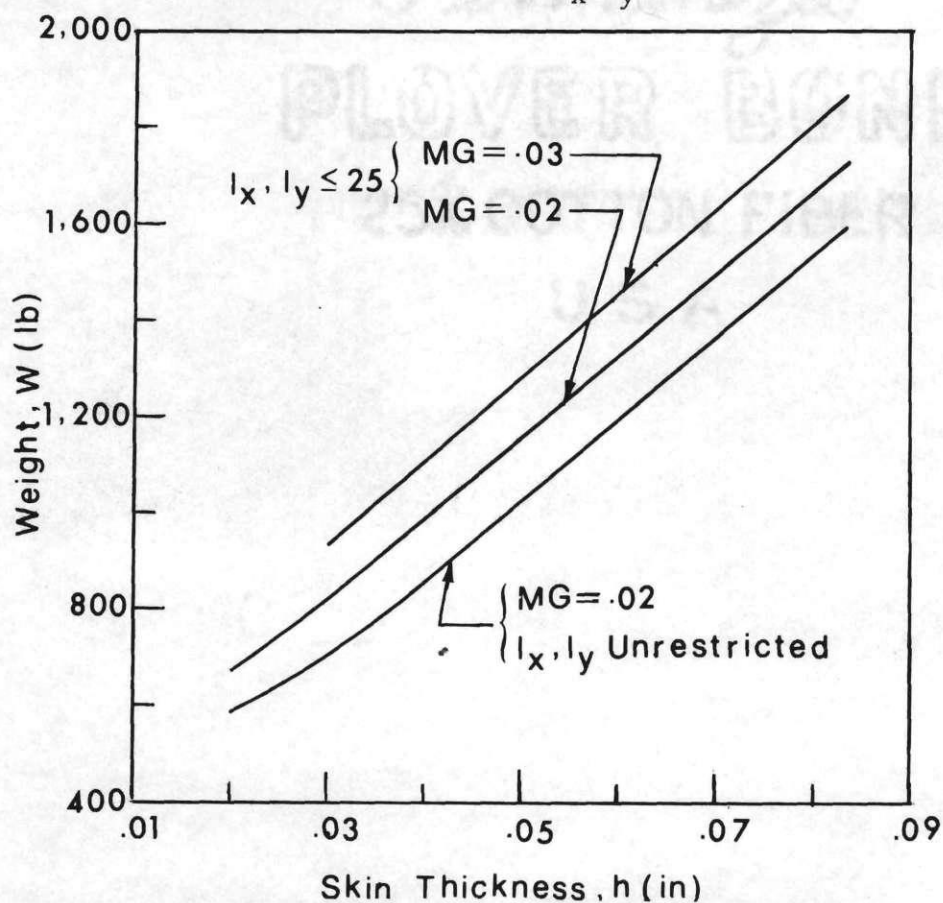


Figure 11: Effect of Gauge Variation and Relaxing Maximum l_x, l_y Limits on the Minimum Weight Design of Example 2 with HS-RR.

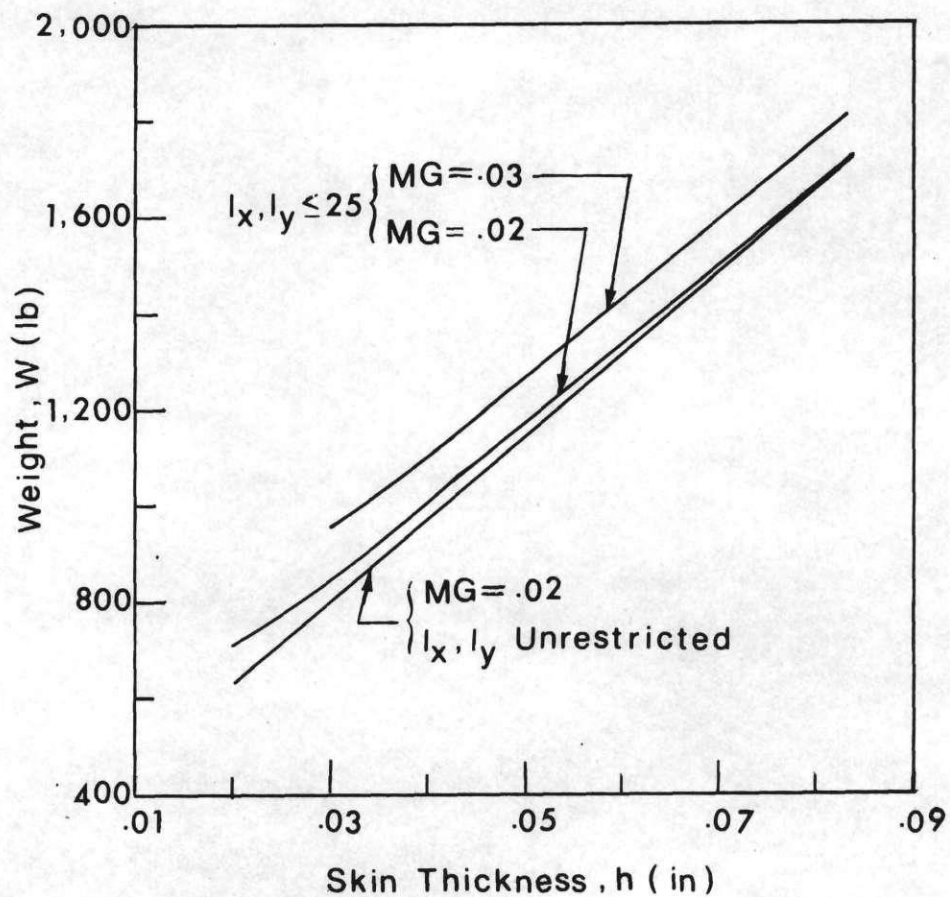


Figure 12: Effect of Gauge Variation and Relaxing Maximum l_x, l_y Limits on the Minimum Weight Design of Example 2 with TS-RR.

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considered and the results are shown both graphically and in tabulation form. WMG designs are obtained only for RS-RR. The experience gained from the designs of Example 1 is employed in considering fewer stiffener geometries for Example 2. For this example the results are shown in Figures 9 to 12. In order for smear technique to hold true the spacings of the stiffeners must be within certain bounds. To start with, a minimum of three rings is adopted for Example 1, which gives $l_y \leq 25$. In the case of the initial minimum number of stringers, $l_x \leq 25$ is adopted. It is necessary to examine the effect of relaxing the maximum limits on l_x and l_y to see how much gain in weight is obtained this way. A study of the effect of gauge relaxation on the weight of the shell is at the same time pertinent. These two effects have been shown for Example 1 in Figures 6 to 8. Similarly, an upper bound of about 25 was first put on l_x and l_y for Example 2 and the effects of gauge variation and relaxing bounds on l_x and l_y are shown in Figures 11 and 12 for HS-RR and TS-RR. The design results at other points on the plots, but for the minimum weight design points, are not given here in order to save space.

Some of the interesting features of the design results obtained are as follows:

1. A design with a skin thickness equal to the MG used is always more economical than any other thickness. Of all stiffening considered, RS-RR are the most uneconomical. For shorter cylinders, from the point of view of weight savings HS-RR, HS-HR, TS-RR, TS-TR and IS-RR seem to be equally efficient, but this competitiveness is immediately eroded away in favor of HS-RR and HS-HR, when compactness

of design is considered, thereby leaving more space inside the fuselage. This fact becomes obvious from a dimensional analysis in Table 2. For a longer shell, Example 2, the HS-RR design, proves to be the most economical from the point of view of weight as well as compactness of design.

2. As the upper bounds on l_x and l_y is decreased, the effect of gauge variation on the weight of the composite cylinder becomes more pronounced. When this bound is completely relaxed, then this effect becomes hardly noticeable for smaller gauge variations. A study of the MG variation and bounds on l_x and l_y greatly helps the designer in estimating the percentage of weight savings.

3. It is observed that the minimum weight design, produced herein, is not unique. There are many combinations of stringer and ring parameters to yield a minimum weight design with nearly same weight. Also, the design that uses both stringers and rings is always more economical than the ones using either stringers alone or rings alone.

4. The distribution of materials in the skin, stringers and rings depends upon (i) the type of stiffeners, (ii) whether the cylinder is short or long and (iii) the skin thickness. On an average it is found that when the final minimum weight design is achieved, the percentage of materials in the skin, stringers and rings varies from 65%, 25% and 10% respectively for Example 1 to 47%, 35% and 18% respectively for Example 2.

Torsion Combined with Axial Compression (without lateral pressure)

For this combined load case, Example 3, the minimum weight design

is first obtained with RS-RR by using a $MG = 0.05$. The effect of reducing the thickness to as less as .03701 in. with a MG of .05 in. for all other dimensions on the weight of the cylinder is studied. A design without minimum gauge, WMG , is also obtained with RS-RR. These results are shown in Table 6 and Figure 13. The best shape and size of stringers and rings are found for this problem as explained below. In order to get a realistic and feasible design, from the manufacturing point of view, all further designs are obtained by fixing MG .

By adopting RR, all open-section types and a closed hat type stringer shapes are examined for weight reduction with a $MG = 0.05$. These results are presented in Tables 7 and 8 and plotted in Figures 14 and 15. As is obvious, TS (or IAS) with $C_x = 1.079$ gives the minimum weight. Next, by adopting the most efficient size and shape of the stringer (TS: $k_x = .3$, $C_x = 1.079$) many designs utilizing open-section types and closed hat type ring shapes are generated keeping the minimum gauge (0.05) fixed. These results are presented in Tables 9 and 10 and in Figures 16 and 17. Comparison of these results leads to the fact that TS-RR ($k_x = .3$, $C_x = 1.079$, $C_y = 1.0$) is the most economical combination of stringer-ring size and shape for the shell of Example 3.

Once the most economical size and shape of stiffeners are determined, the effect of the variation in MG on the shell weight is examined by redesigning the shell for $MG = 0.04$ and 0.03 . Only the final minimum weight design results are given in Table 11 in order to save space. However, in Figure 18 these results are fully demonstrated. The broken lines in Figure 18 simply represent that the design has been

Permitted

Table 6. Design Results for Example 3 with RS-RR.
(Results at $h = .07$ not included in the table)

	WMG				MG = .05		h < .05 $t_x, t_y \geq .05$	
W	390.01553	364.37598	365.43265	433.07775	518.23111	507.32845	498.34946	634.40731
h	.06003	.05000	.04499	.03701	.06003	.05000	.04499	.03701
t_x	.00217	.01163	.02127	.04452	.07156	.09891	.06732	.07638
t_y	.00012	.00031	.00011	.00162	.05131	.05317	.05152	.05104
d_x	2.40120	1.50000	1.34970	.74020	.66033	.50002	.67485	.74020
d_y	1.80090	2.50000	4.27405	4.07110	2.28114	3.50000	2.92435	2.22060
l_x	1.66897	1.40545	1.36560	1.06814	1.94208	1.59424	1.40545	1.21380
l_y	.02499	.15360	.30303	.53475	10.00000	14.28571	10.00000	3.33333
GB	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
PB	.00000	.00001	.00004	.00035	.31792	.95407	.14179	.00909
SB	.00077	.04128	.17492	.56194	.89084	.91081	.92356	.97702
STWB	.57038	.76698	.83385	.47709	.67392	.21218	.86349	.66442
SY	.91222	.87471	.95347	.94513	.73169	.63432	.81871	.79822
STYC	.93680	.95324	.90604	.87042	.70213	.73011	.76337	.70048
RYT	.27224	.28826	.28915	.22115	.19740	.20062	.19402	.12708
n	9	8	7	6	8	7	7	7
n_p	10643	1732	878	498	66	49	27	80

Table 7. Effect of Using Different Open-Section Types of Stringers
for Example 3 with $RR (C_y = 1.0)$ and $MG = .05$.

Type of Stringer	TS or IAS			CS, ZS or IS			AS		RS
k_x	.800	.500	.300	.500	.200	.100	.250	.100	.000
c_x	1.269	1.155	1.079	.707	.798	.866	.707	.845	1.000
w	522.87432	497.16861	486.20235	553.52375	506.22535	497.06073	553.52375	507.34463	507.32845
h	.05000	.05000	.05000	.05000	.05000	.05000	.05000	.05000	.05000
t_{wx}, t_{fx}	.05011	.05041	.05012	.05292	.05052	.05012	.05825	.06466	.09891
t_{wy}	.05027	.05294	.05163	.05441	.05054	.05300	.05441	.05529	.05317
d_{wx}	.54898	.54130	.65738	.53036	.59833	.64956	.66295	.69729	.50002
b_{fx}	.43918	.27035	.19721	.26518	.11967	.06496	.16574	.06973	----
d_{wy}	2.25000	2.75000	2.37500	2.75000	3.00000	3.00000	2.75000	2.75000	3.50000
l_x	1.58950	1.48353	1.60865	1.55252	1.40545	1.58478	1.33518	1.63324	1.59424
l_y	7.14286	10.00000	9.09091	9.09091	11.11111	9.09091	9.09091	11.11111	14.28571
GB	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
PB	.03820	.10178	.04801	.07280	.33768	.08602	.07280	.25349	.95407
SB	.15779	.82086	.95428	.81679	.69886	.97195	.60434	.64366	.91081
STWB	.10175	.10167	.15372	.05119	.08231	.10987	.83345	.79676	.21218
STFB	.12540	.04132	.01889	.04905	.01172	.00377	----	----	----
SY	.77290	.80256	.81200	.73543	.78184	.82633	.73543	.78045	.63432

(Continued)

Table 7. Effect of Using Different Open-Section Types of Stringers
for Example 3 with RR ($C_y = 1.0$) and MG = .05. (Continued)

Type of Stringer	TS or IAS			CS, ZS or IS			AS	RS	
STYC	.72775	.76073	.77088	.68535	.73711	.78831	.68535	.73555	.73011
RYT	.18728	.19931	.20508	.17486	.19564	.19831	.17486	.19513	.20062
n	8	7	8	7	7	7	7	7	7
n _p	38	27	30	30	35	30	30	48	49

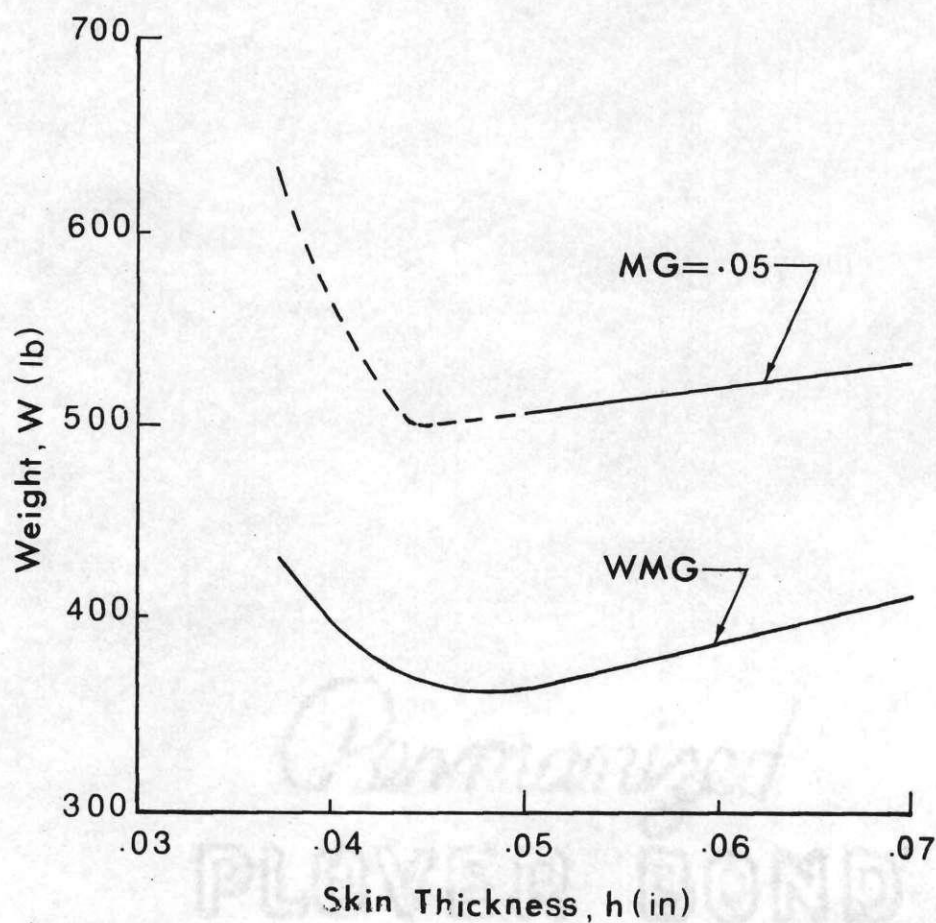


Figure 13: Minimum Weight Design of Example 3 with RS-RR.

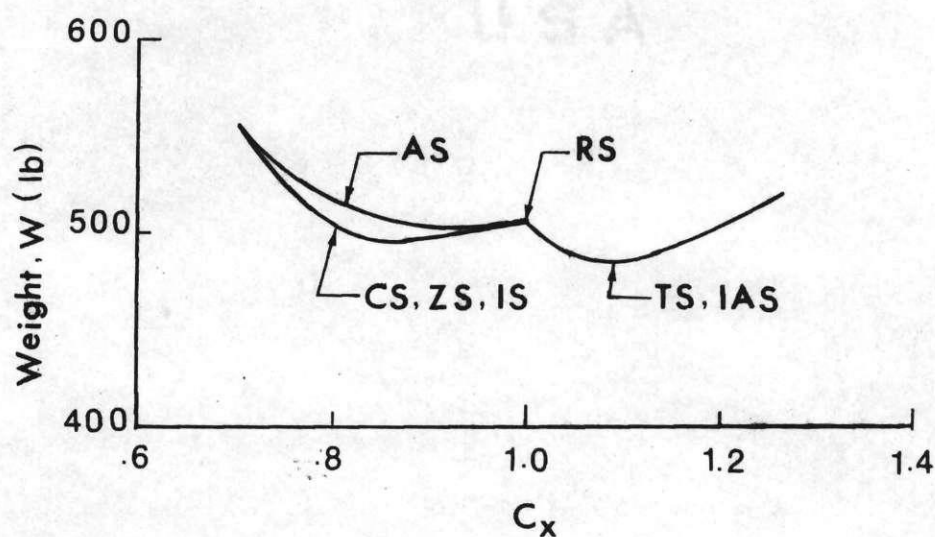


Figure 14: Effect of Stringer Shapes (Open-Section Type) on the Cylinder Weight Using $RR(C_y = 1.0)$ for Example 3.

Table 8. Effect of Using HS for Example 3
with RR ($C_y = 1.0$) and MG = .05.

k_{x1}	.200	.500	.600	1.000
k_{x2}	.000	.000	.000	.000
c_x	.993	.973	.967	.943
w	541.89401	528.17405	529.27715	544.61265
h	.05000	.05000	.05000	.05000
t_{wx}, t_{fx}	.05402	.05005	.05005	.05004
t_{wy}	.05040	.05429	.06120	.05018
d_{wx}	.56901	.50697	.49088	.44197
b_{fx1}	.11380	.25349	.29453	.44197
b_{fx2}	.00000	.00000	.00000	.00000
d_{wy}	2.75000	3.00000	3.00000	2.87500
l_x	1.84800	1.90740	1.87393	1.81657
l_y	10.00000	11.11111	11.11111	10.00000
GB	1.00000	1.00000	1.00000	1.00000
PB	.14914	.27144	.24306	.12360
SB	.86027	.95607	.83551	.63724
STWB	.10253	.08453	.07851	.06185
STFB	.00410	.02113	.02826	.06185
SY	.73430	.75739	.75181	.73430
STYC	.68322	.70982	.70326	.68346
RYT	.18080	.18571	.18564	.17940
n	7	7	7	7
n_p	73	32	30	55

Table 9. Effect of Using Different Open-Section Type of Rings
for Example 3 with Most Efficient Stringer
(TS: $k_x = .3$, $c_x = 1.079$) and MG = .05.

Type of Ring	TR or IAR		CR, ZR or IR		AR	RR
k_y	.600	.300	.200	.100	.14285	.300
c_y	1.193	1.079	.798	.866	.798	1.079
w	497.96694	488.00129	502.69219	496.16800	522.90669	486.20235
h	.05000	.05000	.05000	.05000	.05000	.05000
t_{wx}, t_{fx}	.05008	.05014	.05004	.05000	.05004	.05012
t_{wy}, t_{fy}	.05036	.05032	.05432	.05138	.05362	.05163
d_{wx}	.65738	.61356	.65738	.56973	.54782	.65738
b_{fx}	.19721	.18407	.19721	.17092	.16435	.19721
d_{wy}	1.73545	2.14734	1.99400	2.64130	2.73507	2.37500
b_{fy}	1.04127	.64420	.39880	.26413	.39070	---
l_x	1.58950	1.59901	1.61351	1.35551	1.18157	1.60865
l_y	9.09091	9.09091	9.09091	11.11111	10.00000	9.09091
GB	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
PB	.04744	.06398	.04828	.29962	.17546	.04801
SB	.95004	.98425	.98443	.68788	.50598	.95428
STWB	.22313	.13657	.15418	.11503	.10229	.15372

(Continued)

Table 9. Effect of Using Different Open-Section Type of Rings
for Example 3 with Most Efficient Stringer
(TS: $k_x = .3$, $c_x = 1.079$) and MG = .05. (Continued)

Type of Ring	TR or IAR		CR, ZR or IR		AR	RR
STFB	.15785	.01598	.01898	.01272	.01093	.01889
SY	.80769	.82483	.81026	.80494	.77950	.81200
STYC	.76680	.78597	.77006	.76342	.73551	.77088
RYT	.19857	.20335	.19588	.19973	.18684	.20508
n	8	7	8	7	7	8
n _p	30	30	30	35	75	30

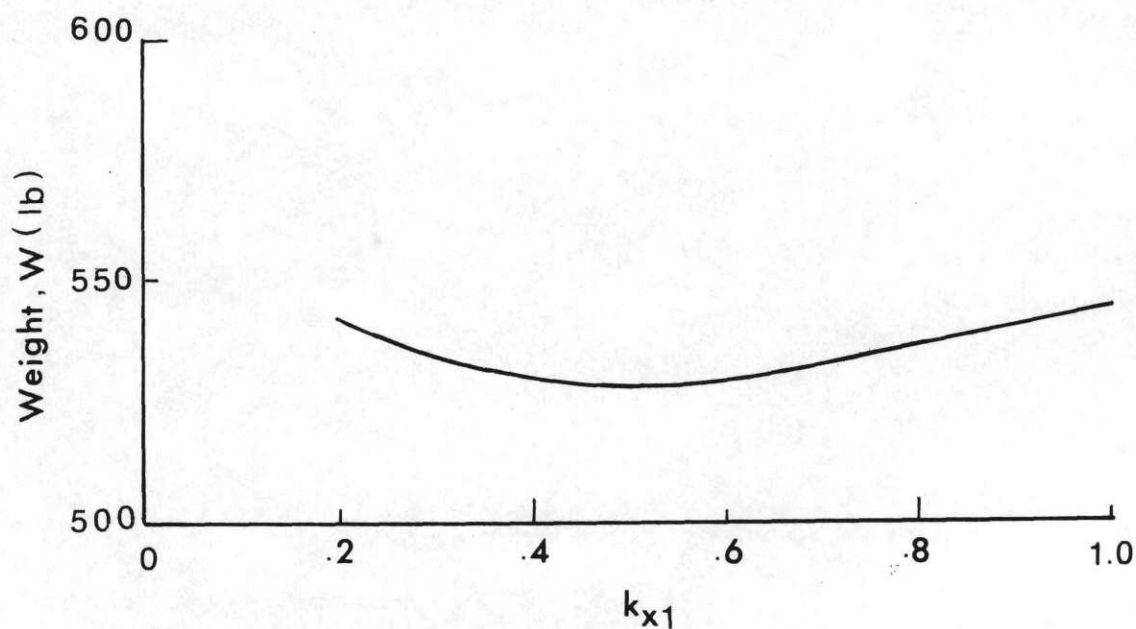


Figure 15: Effect of Using Hat Stringer ($k_{x2} = 0$) with $RR(C_y = 1.0)$ on the Cylinder Weight for Example 3.

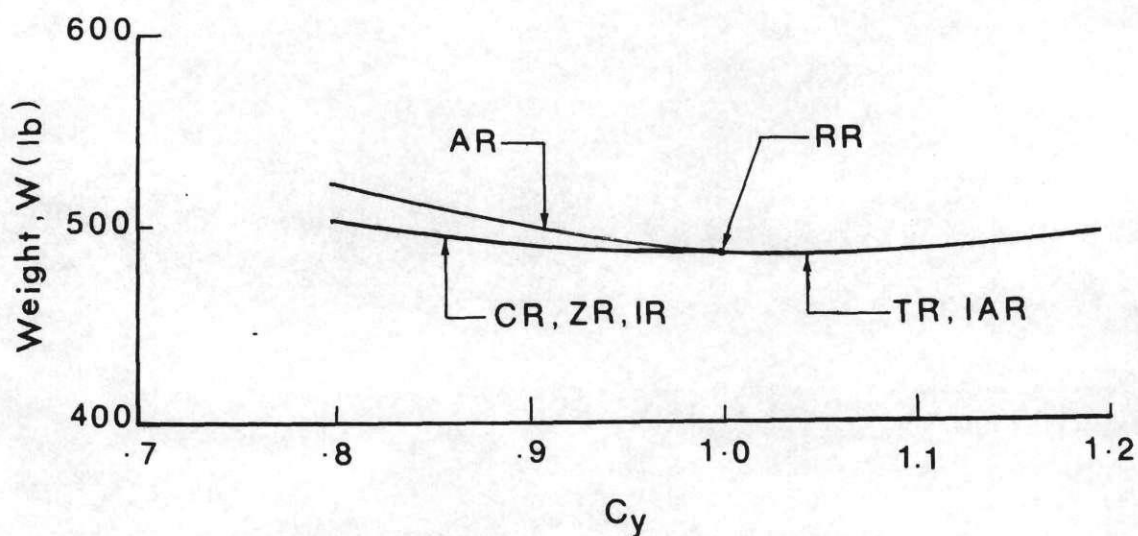


Figure 16: Effect of Ring Shapes on the Cylinder Weight Using Most Efficient Stringer, (TS: $k_x = .3$, $C_x = 1.079$) for Example 3.

Table 10. Effect of Using HR for Example 3
with Most Efficient Stringer
(TS: $k_x = .3$, $c_x = 1.079$) and
MG = .05.

k_{y1}	.200	.500	.800	1.000
k_{y2}	.000	.000	.000	.000
c_y	.993	.973	.954	.943
w	573.61148	518.92043	513.25659	518.91773
h	.05000	.05000	.05000	.05000
t_{wx}, t_{fx}	.05414	.05251	.05000	.05010
t_{wy}, t_{fy}	.05155	.05324	.05326	.05063
d_{wx}	.65738	.65738	.65738	.65738
b_{fx}	.19721	.19721	.19721	.19721
d_{wy}	1.59285	1.62200	1.48360	1.37865
b_{fy1}	.31857	.81100	1.18688	1.37865
b_{fy2}	.00000	.00000	.00000	.00000
l_x	1.33518	1.55253	1.55706	1.56619
l_y	8.33333	12.50000	12.50000	11.11111
GB	1.00000	1.00000	1.00000	1.00000
PB	.02828	.16097	.26851	.09215
SB	.61537	.88648	.90735	.91933
STWB	.11884	.13580	.15256	.15200
STFB	.01382	.01616	.01879	.01870
SY	.74245	.78928	.80128	.80128
STYC	.69471	.74671	.76033	.76066
RYT	.16535	.18835	.19071	.18786
n	8	8	8	8
n_p	32	52	41	24

Table 11. Final Minimum Weight Design Results of Example 3 with the Most Efficient Stiffeners (TS-RR: $k_x = .3$, $c_x = 1.079$, $c_y = 1.0$) and Varying MG (Designs at Other Points not Shown to Save Space).

	MG = .05	MG = .04	MG = .03
w	486.20235	447.66938	414.59044
h	.05000	.04000	.04000
t_{wx}, t_{fx}	.05012	.04000	.03005
t_y	.05163	.04127	.03175
d_{wx}	.65738	.61361	.70127
b_{fx}	.19721	.18408	.21038
d_y	2.37500	3.80000	3.40000
l_x	1.60865	1.16864	1.05132
l_y	9.09091	10.00000	10.00000
GB	1.00000	1.00000	1.00000
PB	.04801	.09501	.06154
SB	.95428	.92560	.75998
STWB	.15372	.23885	.56565
STFB	.01889	.03234	.09187
SY	.81200	.93422	.95422
STYC	.77088	.87484	.89526
RYT	.20508	.21395	.23815
n	8	6	7
n_p	30	27	27

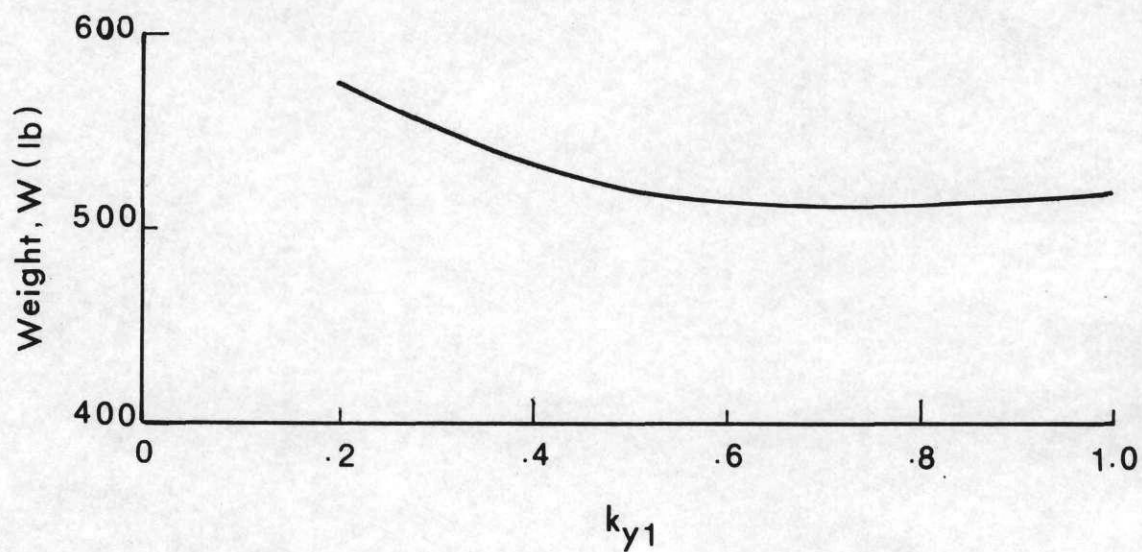


Figure 17: Effect of Using Hat Ring ($k_{y2} = 0$) with the Most Efficient Stringer (TS: $k_x = .3$, $C_x = 1.079$) on the Cylinder Weight for Example 3.

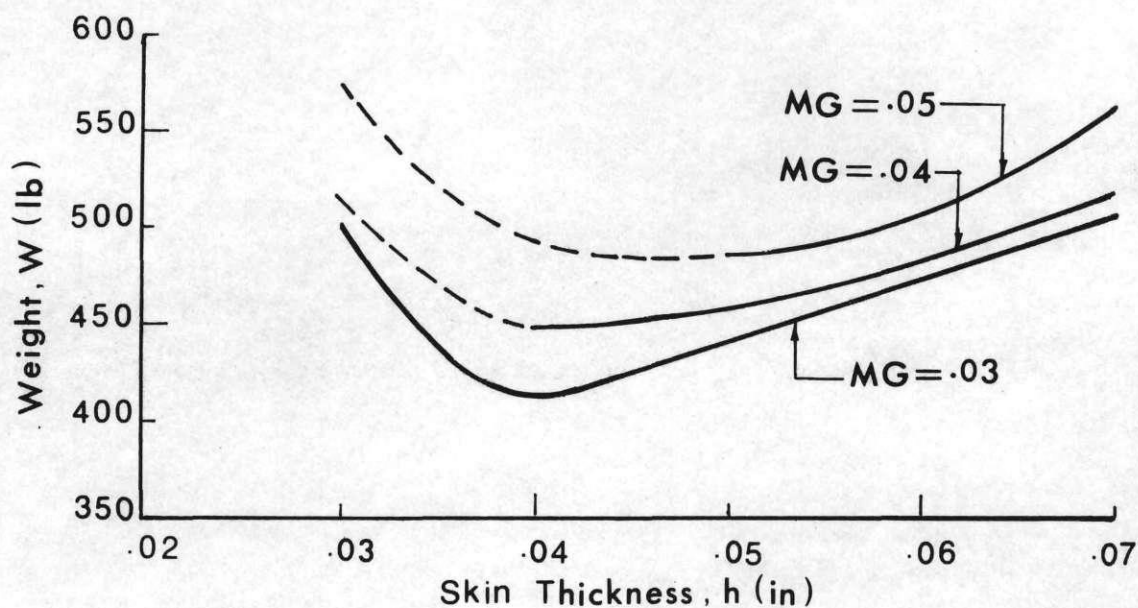


Figure 18: Final Minimum Weight Design of Example 3 Using the Most Efficient Stiffeners (TS-RR: $k_x = .3$, $C_x = 1.079$, $C_y = 1.0$) and Varying MG.

extended to a shell thickness less than that bounded by the specified MG.

Some of the interesting results are listed below.

1. The effect of axial compressive load is predominant for this problem. This is the reason why a closed hat type of stiffener is not more economical.

2. A comparison of results in Tables 7 and 10 shows that the best TS-HR produces a shell with only 5.6 percent more weight than the best TS-RR. This difference is not much and the advantage of using TS-RR can be very well outweighed by TS-HR if the torsional load is comparatively more.

3. It is seen from Figure 14 that the weight savings realized with TS over that with CS, ZS or IS, though marked, is not very large. For such marginal differences other factors, such as availability of material, machining costs, etc., are worth considering.

4. The curves in Figure 16 are quite flat. Nevertheless, RR corresponds to the least weight.

5. As can be seen from Figure 18, the difference in weight because of gauge variation becomes more pronounced at higher values of MG and reduces with lower values.

6. The effect of stringer shape on the cylinder weight is more pronounced than that of the ring shape.

7. Once again it is confirmed that the minimum weight design is not unique and the use of both stringer and ring is more economical than using either alone.

8. While applying a search technique for a combined load case one has to be careful to see that he does not fall in a no-solution zone

at any stage. One way to avoid this trouble, as done in this work, is to start with a value of $\bar{\lambda}_{xx}$ and $\bar{\lambda}_{yy}$ higher than that required to withstand the corresponding compressive load alone.

9. A simple comparison of the three plots in Figure 18 shows that with a MG of 0.05 the most economical design is obtained with a skin thickness between 0.045 and 0.05. By decreasing the MG below 0.05, the minimum weight design corresponds to a skin thickness of about 0.04.

10. When the minimum weight design is achieved with the most efficient stiffeners (TS-RR: $k_x = .3$, $C_x = 1.079$, $C_y = 1.0$), the percentage of weight in the skin, stringers and rings are 55.5, 29.5 and 15.0 respectively; a majority of the weight being contributed by the skin.

Torsion Combined with Axial Compression and Lateral Pressure

This combined load case, Example 4, is taken as a test case. Instead of unnecessarily wasting computer time by analyzing all the stiffener combinations once again, the type of stiffening (TS-RR: $k_x = .3$, $C_x = 1.079$, $C_y = 1.0$) which proved best for Example 3 is considered and the shell is designed for minimum weight. The design results of Examples 3 and 4 with a MG = .05 are given in Table 12 and shown graphically in Figure 19. It is not claimed that the best size and shape of stiffeners of Example 3 will prove to be the best for Example 4 as well. But the contention is that with the best suited stiffeners, if different from that of Example 3, the final minimum weight design will be obtained with a still lower weight.

The most important results are listed below.

Table 12. Design Results of Example 4 and Example 3
with the Most Efficient Stiffeners of Example 3
(TS-RR: $k_x = .3$, $c_x = 1.079$, $c_y = 1.0$) and $MG = .05$.

	Example 4			Example 3		
w	489.94826	436.90637	425.90679	563.99949	507.55836	486.20235
h	.07002	.06003	.05000	.07002	.06003	.05000
t_{wx}, t_{fx}	.05012	.05008	.05006	.05170	.05007	.05012
t_y	.05149	.05207	.05232	.05011	.05163	.05163
d_{wx}	.46029	.52714	.54782	.61372	.57876	.65738
b_{fx}	.13809	.15784	.16435	.18412	.17363	.19721
d_y	1.75050	1.50075	1.25000	1.75050	2.58129	2.37500
l_x	2.54320	2.60522	1.92805	2.10264	1.94208	1.60865
l_y	10.00000	10.00000	6.25000	5.88235	9.09091	9.09091
GB	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
PB	.21431	.13970	.03163	.03864	.10261	.04801
SB	.15944	.36409	.26828	.71613	.94233	.95428
STWB	.06429	.09409	.10774	.10778	.11520	.15372
STFB	.00576	.00969	.01151	.01232	.01289	.01889

(Continued)

Table 12. Design Results of Example 4 and Example 3
 with the Most Efficient Stiffeners of Example 3
 (TS-RR: $k_x = .3$, $c_x = 1,079$, $c_y = 1.0$) and $MG = .05$. (Continued)

	Example 4			Example 3		
SY	.78656	.89018	.95830	.67924	.76800	.81200
STYC	.65704	.73471	.77515	.65926	.74308	.77088
RYT	.51148	.58657	.63262	.18284	.20137	.20508
n	10	11	10	9	8	8
n _p	21	28	43	129	30	30

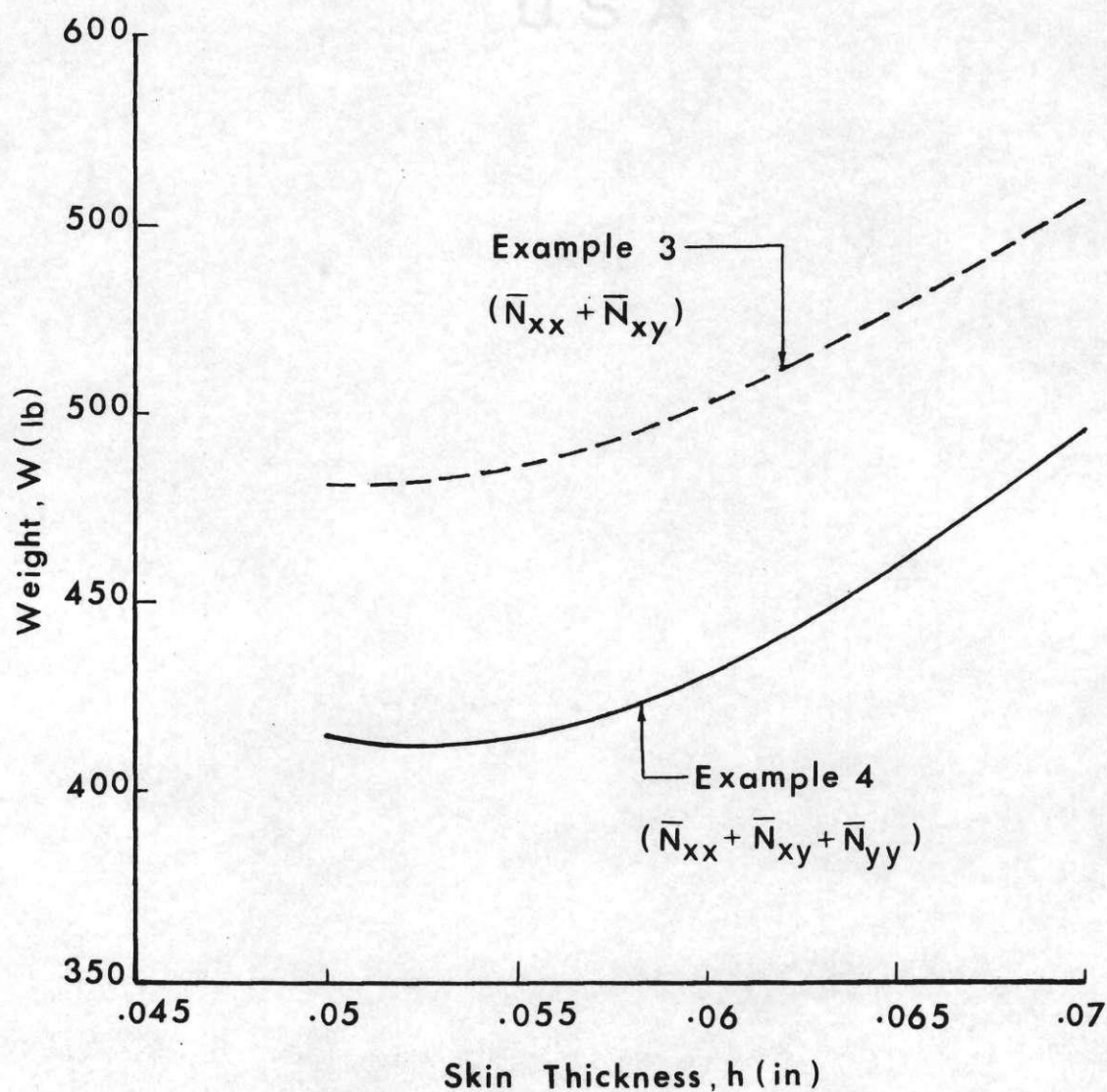


Figure 19: Minimum Weight Design of Examples 4 and 3 Using the Most Efficient Stiffeners of Example 3 (TS-RR: $k_x = .3$, $C_x = 1.079$, $C_y = 1.0$) and $MG = .05$.

1. A simple comparison of the design results (Table 12 or Figure 19) shows that internal pressure is greatly helpful in reducing the weight (for a fuselage type of structure, external pressure is not a question and hence not attempted).

2. It can very well be inferred that torsion combined with axial compression (without lateral pressure) is characteristic of the worst possible loading case for fuselage type of structures.

3. A comparison of the results of Examples 3 and 4 in Table 12 shows that skin buckling, which is an active mode of failure for the two load case ($\bar{N}_{xx} + \bar{N}_{xy}$), Example 3, is no longer active for the three load case ($\bar{N}_{xx} + \bar{N}_{xy} + \bar{N}_{yy}$), Example 4.

4. With pressure loading the rings become more effective in sharing the load. This can be seen by comparing the values of RYT (ring yielding in tension) for the two examples.

5. If not for all types of stiffeners, at least for the case considered it is concluded that the use of both stringers and rings leads to a more efficient configuration over using either stringers alone or rings alone.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The most important conclusions based on the Examples considered in the present work are listed below.

Pure Torsional Load

1. From the point of view of weight savings and compactness of design, HS-RR and HS-HR are more economical for short as well as long cylinders.
2. Invariably the minimum weight design at the preset MG for the skin thickness, h is the most economical for all types of stiffeners.
3. The upper bounds on l_x and l_y , from consideration of the applicability of the smeared technique, are more important factors in arriving at the minimum weight design for most types of stiffeners.
4. The higher the bounds on l_x and l_y the lower the effect of gauge variation.

Torsion Combined with Axial Compression (Without Lateral Pressure)

1. The effect of stringer geometry on the weight of the cylinder is more pronounced than that of the ring geometry.
2. When the axial compression is predominant as compared to the torsion then, at least for short cylinders, open-section type stiffeners (TS-RR) are more efficient.
3. It cannot be inferred that one particular shape and size of

stiffeners will be best for all combined load problems. Each problem should be solved separately.

4. In order to obtain a correct solution, it is necessary to see that the designer does not fall in a no-solution zone at any stage. This can be done by starting and maintaining the values of the independent variables ($\bar{\lambda}_{xx}$, $\bar{\lambda}_{yy}$ here) at a higher level than that required for the corresponding compressive load alone, which, in turn, requires a solution for the compressive load first.

5. The difference in weight because of gauge variation is more pronounced at higher values of MG.

6. The skin thickness which corresponds to the minimum weight design and the cylinder weight depend upon the MG adopted. This shows that a decision about MG is an essential factor.

Torsion Combined with Axial Compression and Lateral Pressure

1. Internal pressure is greatly helpful in reducing the shell weight.

2. With internal pressure loading the rings become more active in sharing the load and skin buckling ceases to be an active mode of failure, for a pressure of one atmosphere.

General (For All Loading Cases)

1. Stiffening in both directions, circumferential and longitudinal, produces a more economical design than any one alone.

2. The minimum weight design is not unique. There exist possibilities of adjusting the parameters to arrive at a final design of nearly the same weight. Furthermore, non-uniqueness of optimum implies flexibility to deal in the future with imperfection

sensitivity and cost trade-offs.

3. An automated design in five stages, as produced herein for the combined load case, highly facilitates the designer in arriving at a minimum weight design with reduced computer time.

4. The present approach gives full freedom and complete control to the designer in making a decision for the penalty in weight that he is forced to pay in order to relax some of the constraints and requirements. Additionally, failure mode interaction can be avoided easily and the active failure modes are wisely separated out to the desired extent.

A Comparison with the Results of Axial Compression

A comparison of the results of minimum weight design of the same cylinder (C-141 fuselage) to withstand only axial compression (see Reference [20]) with that of torsion combined with axial compression shows the following salient points.

1. By adding torsion over the existing axial compressive load the additional weight required is 13.202 lb. over 473.0 lb., i.e., an increase of 2.791 per cent.

2. The optimizing size and shape of stiffeners (TS-RR: $k_x = 0.3$, $C_x = 1.079$, $C_y = 1.0$) are the same for both loading cases. This shows that the predominant axial compressive load is more instrumental in influencing the stiffener geometries.

3. There is no appreciable difference in the stringer depth and width but the ring depth is appreciably increased from 1.75 inch for simple compression to 2.375 inch for torsion combined with axial compression, a 35.714 per cent increase.

4. Stringer spacing is very nearly the same for the two loading cases but the ring spacing is reduced appreciably (from 12.5 inches for compression only to 9.091 inches for torsion combined with axial compression).

5. Panel buckling is an active failure mode for simple compression ($PB = 0.8821$) which is not the case for torsion with axial compression ($PB = 0.048$).

6. The percentage of weight for a simple compression, see Reference [20], in the skin, stringers and rings are 60.0, 30.0 and 10.0 respectively. For a combined load case this changes to 55.5, 29.5 and 15.0 respectively, showing a transfer of some skin material to the rings with the stringer material remaining more or less the same.

7. Unless one solves a number of problems, no generalization can be made, by deriving a logical conclusion from the above mentioned (Numbers one to six) observations of one shell geometry, which could be said to be valid for all types of cylinders and loadings.

Recommendations

1. The method adopted herein can be used for shells other than circular cylindrical.

2. Other closed type stiffener shapes, such as inverted V and Y, should be examined for any further weight reduction.

3. The present method should be extended to the truss-core sandwich type of cylinders and to the cylinders with stiffeners not aligned in the circumferential and longitudinal directions.

4. A composite shell should also be analyzed before arriving at a decision as to which type, out of conventionally stiffened, truss-core type and composite shell, is most efficient.

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APPENDICES

APPENDIX A

ASSUMPTIONS IN THE STIFFENED CYLINDRICAL SHELL ANALYSIS

1. The shell is thin.
2. The deflections are small.
3. The rotations about the inplane axes are very large in comparison to the rotation about the normal axis.
4. Normals to the reference surface before deformation remain normal even after deformation and they are inextensional, so that
$$\gamma_{xz} = \gamma_{yz} = \epsilon_{zz} = 0.$$
5. The stiffeners are along the principal curvatures.
6. Smear technique holds true, i.e., the spacings of the stiffeners are close enough so that the flexural and extensional stiffness are distributed mathematically over the whole surface of the shell.
7. The stiffener-to-skin connection is monolithic.
8. Opened type of stiffeners are torsionally weak.
9. Stiffeners are in the uniaxial stress state.
10. The membrane shear force is carried entirely by the skin.

APPENDIX B

PROPERTIES OF STIFFENERS

With the understanding that there will be no summation on i and with i representing either x or y which in turn represent respectively the stringer and the ring, it can be easily shown that for open as well as close type of stiffeners when $t_f, t_w \ll d$

$$\text{Nondimensional radius of gyration, } \bar{\alpha}_i = p_i \left(\frac{d_i}{h} \right)$$

$$\text{Nondimensional flexural stiffness, } \bar{\rho}_{ii} = \bar{\alpha}_i^{-2} \bar{\lambda}_{ii}$$

$$\text{Nondimensional stiffener eccentricity, } \bar{e}_i = \frac{\pi^2 (1-\nu^2)^{\frac{1}{2}}}{2Z} (1 + C_i \bar{\alpha}_i)$$

where the values of the parameters p_i and C_i depend upon the type of stiffener geometry selected.

(i) Open Stiffeners:

With the notations as in Figure B1 and starting with the basic definition, i.e., the radius of gyration of stiffener, $\alpha = [I/A]^{\frac{1}{2}}$ and the radius of gyration of skin per unit length $= h/\sqrt{12}$, p_i and C_i for the commonly used open types of stiffeners are given in Table B1.

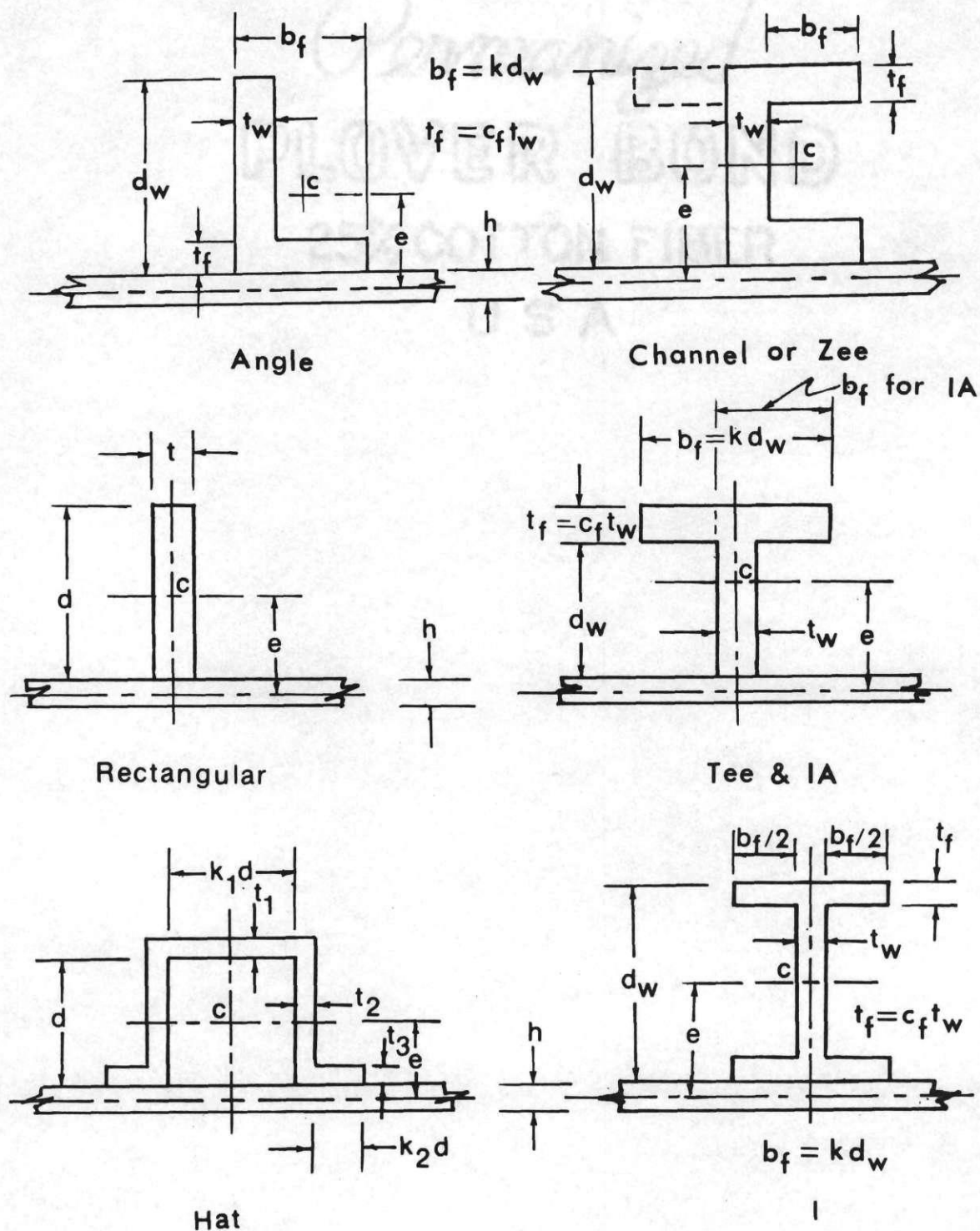


Figure B1: Stiffener Geometry

Table B1. Properties of Open-Section Type of Stiffeners.

Type of Stiffener	p_i	C_i
Rectangular	1.0	1.0
Tee, Inverted Angle	$\frac{(1 + 4c_{fi} k_i)^{1/2}}{1 + c_{fi} k_i}$	$\frac{1 + 2c_{fi} k_i}{(1 + 4c_{fi} k_i)^{1/2}}$
Channel, I, Z	$\left(\frac{1 + 6c_{fi} k_i}{1 + 2c_{fi} k_i}\right)^{1/2}$	$\left(\frac{1 + 2c_{fi} k_i}{1 + 6c_{fi} k_i}\right)^{1/2}$
Angle	$\frac{(1 + 4c_{fi} k_i)^{1/2}}{1 + c_{fi} k_i}$	$\frac{1}{(1 + 4c_{fi} k_i)^{1/2}}$

(ii) Closed (Hat) Stiffener: With the notations in Figure B1 and assuming that $t_{1i} = t_{2i} = t_{3i} = t_i \leq d_i$ and $t_i \sim h$

$$\text{Area, } A_i = d_i t_i (2 + k_{1i} + 2k_{2i})$$

$$\text{M.I., } I_{xc} = \frac{d_i^4}{12} \left(\frac{t_i}{d_i} \right) \left(\frac{6k_{1i}^2 + 8k_{1i} + 16k_{2i} + 24k_{2i}^2 + 4}{2 + k_{1i} + 2k_{2i}} \right)$$

$$\text{M.I., } I_{yc} = \frac{d_i^4}{12} \left(\frac{t_i}{d_i} \right) [6k_{1i}^2 + (k_{1i} + 2k_{2i})^3]$$

$$\frac{(GJ)_i}{2D\ell_i} = \frac{3\bar{\lambda}_{ii}}{1+\nu} \frac{J_i}{A_i h^2} = q_i \bar{\lambda}_{ii} \bar{\alpha}_i^2$$

$$J_i = \frac{4A^2}{\int_0^s \frac{ds}{t}} \text{ for any closed section.}$$

From these we have p_i , C_i and q_i for hat section as shown in Table B2.

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Table B2. Properties of Hat Stiffener.

Type of Stiffener	p_i	c_i	q_i
Hat	$\frac{(6k_{11}^2 + 8k_{11} + 16k_{21} + 24k_{21}^2 + 4)}{k_{11} + 2 + 2k_{21}}$	$\frac{2(k_{11} + 1)}{(2k_{11}^2 + 8k_{11} + 16k_{21} + 24k_{21}^2 + 4)^{\frac{1}{2}}}$	$\frac{12}{(1+\nu)} \frac{k_{11}^2}{(4 + 4k_{11} + 3k_{21})(3k_{11}^2 + 4k_{11} + 8k_{21} + 12k_{21}^2 + 2)}$

APPENDIX C

Z-RANGE FOR GENERATING DATA

Referring to [42] the critical stress for a uniform thickness isotropic cylindrical shell is given by

$$\bar{K}_{s_{cr}} = .925 Z^{3/4}$$

where for our case $\bar{K}_{s_{cr}} = \frac{\bar{N}_{xy} L^2}{\pi^2 D}$, $D = Eh^3/12(1-\nu^2)$ and $Z = \frac{L^2(1-\nu^2)^{1/2}}{Rh}$. For shell problem 1 substituting $\bar{N}_{xy} = 418.538$, $R = 85$, $L = 100$, $E = 10.5 \times 10^6$ and $\nu = .33$ and solving the above equations one gets $Z_{un} \simeq 750$. The higher limit for Z is fixed by selecting a MG for $h = .05$ which fixes $Z_{max} \simeq 2200$, and $Z_{max} \simeq 7000$ for a WMG design from a consideration that yielding in the skin material should not take place. Similarly substituting $\bar{N}_{xy} = 125$, $R = 95.5$, $L = 291$ and E and ν as above for the shell problem 2, we get $Z_{un} = 7,500$. The higher limit of Z for this problem with MG = .02 for h becomes $Z_{max} \simeq 41,000$ and for a WMG design $Z_{max} \simeq 170,000$ for the yielding of the skin not to take place.

Having known the minimum weight design results for pure torsion and for axial compression from Reference [20], it becomes quite simple to decide about the Z -range for the combined load case.

APPENDIX D

SOLUTION OF EIGENVALUES
AND EIGENVECTORS WHEN $\lambda_1 = -\lambda_2$

Consider

$$[A + \bar{K}_s B]\{X\} = 0 \quad (D1)$$

where the lowest value of \bar{K}_s is required and A and B are symmetric matrices and A is positive definite. Then the minimum value, $\bar{K}_{s \min}$ is always real. By putting $C = A^{-1}B$ and $\lambda = \frac{1}{\bar{K}_s}$, Equation (D1) reduces to

$$[C + \lambda I]\{X\} = 0 \quad (D2)$$

Equation (D2) is the usual eigenvalue problem. For $\bar{K}_{s \min}$ we shall have maximum value of λ . Furthermore, if \bar{K}_s is an eigenvalue, so is $-\bar{K}_s$. Besides, from the physics of the problem (torsional loading) the sign of \bar{K}_s (and hence of λ) does not make any difference.

Let $\lambda_1 = -\lambda_2$ and $|\lambda_1| > |\lambda_3| > \dots > |\lambda_n|$. Then an arbitrary vector V can be written as a linear combination of the eigenvectors X_1, X_2, \dots, X_n of -C; Reference [39], [40]. One can write

$$V = \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 + \dots + \alpha_n X_n$$

$$-CV = \alpha_1 \lambda_1 X_1 + \alpha_2 (-\lambda_1) X_2 + \alpha_3 \lambda_3 X_3 + \dots + \alpha_n \lambda_n X_n$$

$$(-C)^m V = \alpha_1 \lambda_1^m X_1 + \alpha_2 (-\lambda_1)^m X_2 + \alpha_3 \lambda_3^m X_3 + \dots + \alpha_n \lambda_n^m X_n$$

For simplicity let $V_1 = -CV$, $V_2 = -CV_1 = (-C)^2 V$, or in general

$V_m = (-C)^m V$. Then $V_{m+1} = -CV_m$. Substituting $C = A^{-1}B$ and

premultiplying by A

$$AV_{m+1} = -BV_m \quad (D3)$$

The lowest eigenvalue, $\bar{K}_{s_{\min}}$ is obtained as follows:

$$V_{2m} = \alpha_1 \lambda_1^{2m} X_1 + \alpha_2 (-\lambda_1)^{2m} X_2 + \cdots + \alpha_n \lambda_n^{2m} X_n$$

$$V_{2m} = \lambda_1^{2m} \left[\alpha_1 X_1 + \alpha_2 X_2 + \cdots + \alpha_n \left(\frac{\lambda_n}{\lambda_1} \right)^{2m} X_n \right]$$

For large m this reduces to

$$V_m = \lambda_1^{2m} (\alpha_1 X_1 + \alpha_2 X_2)$$

similarly

$$V_{2m+2} = \lambda_1^{2m+2} (\alpha_1 X_1 + \alpha_2 X_2)$$

Then

$$\frac{V_{2m+2}^T V_{2m}}{V_{2m+2}^T V_{2m+2}} = \frac{\lambda_1^{4m+2}}{\lambda_1^{4m+4}} = \left(\frac{1}{\lambda_1} \right)^2$$

Remembering that $\bar{K}_s = 1/\lambda_1$, we have

$$\bar{K}_s^2 = \frac{V_{2m+2}^T V_{2m}}{V_{2m+2}^T V_{2m+2}} \quad (D4)$$

The eigenvectors are obtained by the following method

$$V_{2m} = \lambda_1^{2m} \left[\alpha_1 X_1 + \alpha_2 X_2 + \cdots + \alpha_n \left(\frac{\lambda_n}{\lambda_1} \right)^{2m} X_n \right]$$

$$V_{2m+1} = \lambda_1^{2m+1} \left[\alpha_1 X_1 - \alpha_2 X_2 + \dots + \alpha_n \left(\frac{\lambda_n}{\lambda_1} \right)^{2m+1} X_n \right]$$

For large m ,

$$V_{2m+1} + \lambda_1 V_{2m} = 2\alpha_1 \lambda_1^{2m+1} X_1$$

and

$$V_{2m+1} - \lambda_1 V_{2m} = -2\alpha_2 \lambda_1^{2m+1} X_2$$

Since a scalar multiple of the eigenvector represents the eigenvector itself and X_1 and X_2 are the eigenvectors corresponding to λ_1 and $-\lambda_1$

$$\begin{aligned} X_1 &= V_{2m+1} + \lambda_1 V_{2m} && \text{for positive } \lambda_1 \\ X_2 &= V_{2m+1} - \lambda_1 V_{2m} && \text{for negative } \lambda_1 \end{aligned} \tag{D5}$$

Alternatively

$$\begin{aligned} X_1 &= V_{2m} + \bar{K}_s V_{2m+1} && \text{for positive } \bar{K}_s \\ X_2 &= V_{2m} - \bar{K}_s V_{2m+1} && \text{for negative } \bar{K}_s \end{aligned} \tag{D6}$$

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APPENDIX E

ALTERNATIVE APPROACH FOR ESTIMATING $\bar{K}_{s\text{cr}}$ FOR COMBINED LOADING

In employing the flexible polyhedron type of simplex method one has to evaluate $\bar{K}_{s\text{cr}}$ at each vertex of the polyhedron. In effect this requires finding $\bar{K}_{s\text{cr}}$ for a known geometry of the shell. For combined loading, one can simplify this calculation by making use of interaction curve as shown in Figure E1 where points A and B correspond to the critical loads in simple torsion and axial compression respectively, i.e., $\bar{N}_{xy\text{cr}}$ and \bar{N}_{cr} , for a fixed geometry.

As already explained, for a known geometry of the shell $\bar{N}_{xy\text{cr}}$ (A) can be found by finding $\bar{K}_{s\text{cr}}$ after solving a 5×5 determinant for \bar{K}_s and minimizing it w.r.t. $\beta (= nL/\pi R, n \geq 2)$. Also \bar{N}_{cr} (B) can be found by the method outlined in References [19] and [20]. A knockdown factor and a factor of safety on \bar{N}_{cr} and $\bar{N}_{xy\text{cr}}$ can be suitably imposed. The present state of the art dictates a factor of 0.65 for \bar{N}_{cr} and 0.80 for $\bar{N}_{xy\text{cr}}$. This means that the critical condition in the presence of imperfections can be taken as those predicted by linear theory multiplied by the above factors. As an estimate (the usual engineering approach) a straight line (shown dotted in the figure) may be taken as an approximation to replace the exact interaction curve [44]. Having found this straight line interaction for any geometry of the shell, one can easily find $\bar{N}_{xy\text{cr}}$ for any known value of \bar{N} and hence $\bar{K}_{s\text{cr}}$ can be found for the combined loading.

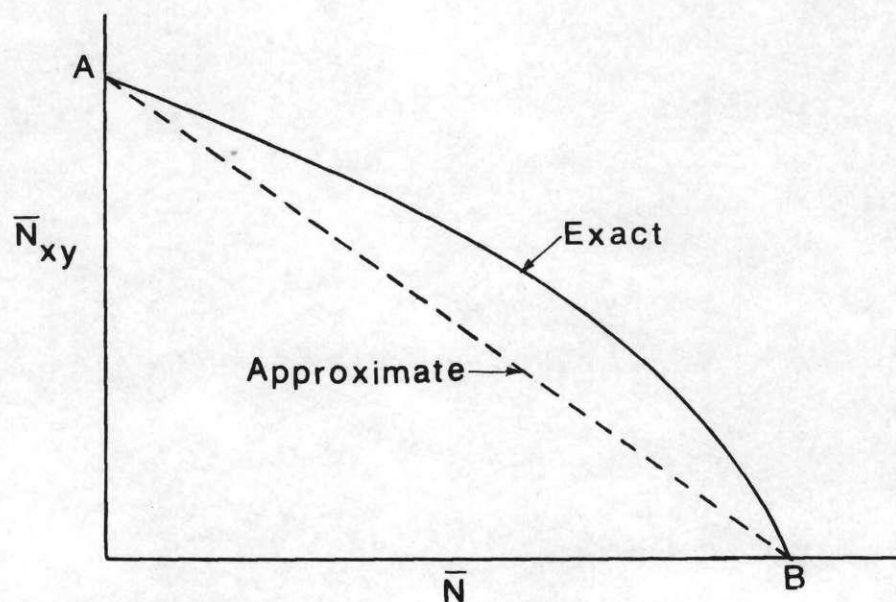


Figure E1: Interaction Curve for
Combined Loading

One should not lose sight of the fact that the exact interaction curve has been approximated by a straight line and the value of $\bar{K}_{s_{cr}}$ depends upon the knockdown factor and the factor of safety assumed by the designer on \bar{N}_{cr} and $\bar{N}_{xy_{cr}}$. Also the procedures explained herein has to be followed and $\bar{K}_{s_{cr}}$ has got to be evaluated at each of the vertices of the polyhedron.

APPENDIX F

SAMPLE DESIGN

With the design procedure discussed in Chapter III one example for pure torsion and another for torsion combined with axial compressive load are solved here for the purpose of illustration. The associated design tables, i.e., the data generated from the program in stages 1 and 2 respectively are also given.

(a) Pure Torsional Load

The shell problem considered is Example 1 of Chapter IV. Type of stiffening is RS - RS and $Z = 2000$. $MG = .05$ in.

The design steps are listed in Chapter III.

$$h = \frac{L^2(1-\nu^2)^{\frac{1}{2}}}{ZR} = .05553$$

From Table F1, $\bar{\alpha}_x = 40$, $\bar{\alpha}_y = 79$

$$\bar{\lambda}_{xx} = .41533, \quad \bar{\lambda}_{yy} = .16921$$

$$\bar{W} = 1.65597, \quad n = 2$$

Calculate the stress in the skin.

$$\sigma_{xy_{sk}} = \frac{\bar{N}_{xy}}{h} = 7,538.50864 \text{ psi.}$$

This stress is less than the yield stress in the skin material.

Table F1. Design Chart for Example 1 with RS-RR.

ν	Z	C_x	C_y	\bar{N}_{xy}^*	
.330	2000.000	1.000	1.000	.31530E-6	
$\bar{\alpha}_x$	$\bar{\alpha}_y$	$\bar{\lambda}_{xx}$	$\bar{\lambda}_{yy}$	\bar{W}	n
20.0	110.0	.47928	.67030	2.29006	2
20.0	130.0	.37932	.57279	2.06847	2
20.0	150.0	.31870	.48462	1.90149	2
30.0	80.0	.63502	.28469	2.03210	2
30.0	90.0	.56502	.29936	1.97001	2
30.0	100.0	.45745	.35477	1.91149	2
30.0	110.0	.41809	.33406	1.84407	2
30.0	120.0	.40000	.29785	1.78313	2
30.0	130.0	.43389	.22704	1.74171	2
30.0	140.0	.30938	.29474	1.67795	2
40.0	60.0	.40947	.31328	1.81107	2
40.0	70.0	.48326	.14226	1.70196	3
40.0	80.0	.56579	.07140	1.71506	3
40.0	90.0	.36924	.20403	1.64333	2
40.0	100.0	.34758	.19620	1.61023	2
40.0	110.0	.38400	.12010	1.56570	2
50.0	20.0	.60556	.24163	1.95073	5
50.0	30.0	.47741	.22137	1.78418	4
50.0	40.0	.46596	.13420	1.67350	4
50.0	50.0	.42256	.11103	1.59880	4
50.0	60.0	.36075	.12850	1.54904	3

(Continued)

Table F1. (Continued)

$\bar{\alpha}_x$	$\bar{\alpha}_y$	$\bar{\lambda}_{xx}$	$\bar{\lambda}_{yy}$	\bar{w}	n
50.0	70.0	.32746	.12439	1.50707	3
60.0	10.0	.66505	.03678	2.01204	8
60.0	20.0	.47585	.16637	1.72071	6
60.0	30.0	.37380	.15031	1.58816	5
60.0	40.0	.30092	.16332	1.52097	4
38.0	77.0	.58504	.09472	1.76283	3
38.0	79.0	.45598	.17467	1.70996	2
38.0	80.0	.46084	.16390	1.70109	2
38.0	83.0	.45523	.16125	1.69182	2
38.0	87.0	.46118	.13810	1.67251	2
38.0	90.0	.44008	.15829	1.67149	2
38.0	93.0	.42934	.16272	1.66442	2
38.0	97.0	.41007	.17440	1.65590	2
38.0	100.0	.44134	.12677	1.63753	2
39.0	72.0	.55838	.10678	1.74644	3
39.0	75.0	.43134	.19925	1.70765	2
39.0	78.0	.44978	.14947	1.67248	2
39.0	82.0	.43924	.15304	1.66466	2
39.0	86.0	.40724	.19091	1.67125	2
39.0	90.0	.44587	.11771	1.63245	2
40.0	72.0	.44869	.15467	1.67934	3
40.0	74.0	.43570	.15671	1.66482	3
40.0	76.0	.45188	.13869	1.66275	3
40.0	78.0	.40744	.18950	1.66989	2
40.0	79.0	.41533	.16921	1.65597	2
40.0	82.0	.43504	.12798	1.63183	3
41.0	66.0	.45739	.16217	1.69505	3

(Continued)

Table F1 (Continued)

$\bar{\alpha}_x$	$\bar{\alpha}_y$	$\bar{\lambda}_{xx}$	$\bar{\lambda}_{yy}$	\bar{w}	n
41.0	68.0	.63784	.14725	1.66904	3
41.0	70.0	.47368	.13307	1.68087	3
41.0	72.0	.44852	.14197	1.66266	3
41.0	74.0	.57643	.06477	1.71956	3
42.0	62.0	.42514	.19361	1.69437	3
42.0	64.0	.47280	.14345	1.68932	3
42.0	66.0	.45957	.14335	1.67660	3
42.0	68.0	.50704	.10276	1.68431	3
42.0	70.0	.49982	.10139	1.67468	3
43.0	64.0	.43111	.15876	1.66195	3
43.0	66.0	.45471	.13060	1.65684	3
43.0	68.0	.40239	.16361	1.63518	3

On the basis of minimum three rings, $l_y = 25$ in.

For $|\sigma_{xy_{sk_{cr}}}| > |\sigma_{xy_{sk}}|$ and $l_y > l_x$ we must have

$$\frac{\pi^2 E}{12(1-\nu^2)} h^2 \left[\frac{5.34}{l_x^2} + \frac{4}{l_y^2} \right] > 7,538.50864. \quad \text{Substituting for } E, h \text{ and } l_y$$

and solving the above, one gets

$$l_x < 4.66053 \quad (F1)$$

Adopting 120 stringers, $l_x = 4.4506083$.

Also for $MG = .05$ on t_x and t_y we must have $\frac{\bar{\lambda}_{xx} l_x}{.8911 \bar{\alpha}_x} > .05$ as well as

$$\frac{\bar{\lambda}_{yy} l_y}{.8911 \bar{\alpha}_y} > .05, \text{ i.e., } l_x > 4.29056 \text{ and } l_y > 20.79930 \quad (F2)$$

which are satisfied.

Check for panel buckling load gives

$$\bar{N}_{xy_p} = 3199.098, \quad n_p = 72$$

$$\therefore PB = 418.538/3199.098 = .13083.$$

With the values of l_x and l_y adopted, one gets

$$\sigma_{xy_{sk_{cr}}} = 8244.41541 \text{ psi}$$

$$SB = \sigma_{xy_{sk}} / \sigma_{xy_{sk_{cr}}} = .91437$$

Finalizing all remaining dimensions,

$$d_x = h \bar{\alpha}_x = 2.22113, \quad d_y = h \bar{\alpha}_y = 4.38673$$

$$t_x = \frac{\bar{\lambda}_{xx} l_x}{.8911 \bar{\alpha}_x} = .05186, \quad t_y = \frac{\bar{\lambda}_{xx} l_y}{.8911 \bar{\alpha}_y} = .06009$$

Weight of the stiffened shell = $2\pi R L h \rho_{sk} \bar{W} = 496.006358$.

It is obvious from the above steps that l_y and l_x can be changed or adjusted within the bounds given by (F1) and (F2) to satisfy all the required constraints on PB and SB. Furthermore, the designer can move to another point in $\bar{\alpha}_x - \bar{\alpha}_y$ space either having the same value of \bar{W} or very nearly same value and still satisfy all the constraints by following the above steps. This amply proves that the minimum weight design is not unique.

(b) Torsion Combined with Axial Compression

The shell problem is Example 3 of Chapter IV. Type of stiffening is TS-RR ($k_x = 0.3$, $C_x = 1.079$, $C_y = 1.0$) and $Z = 2221$. $MG = .05$.

See Chapter III for the design steps.

$$h = \frac{L^2(1-\nu^2)^{\frac{1}{2}}}{ZR} = .05000$$

For thin ring theory to hold true, $\bar{\alpha}_y$ must be less than

$$85 \left[= \frac{R}{20h} \frac{(1+C_{fy} k_y)^{\frac{1}{2}}}{1+C_{fy} k_y} \right].$$

As a large number of data are generated for each point in $\bar{\alpha}_x - \bar{\alpha}_y$ space by the program 'OPTIMUM' of Stage 2 for the design procedure to become highly automated, only few points have been demonstrated in the design chart of Table F2 in order to save space.

From Table F2,

$$\bar{\alpha}_x = 15.0 , \quad \bar{\alpha}_y = 47.5$$

$$\bar{\lambda}_{xx} = .47485 , \quad \bar{\lambda}_{yy} = .24044$$

$$\bar{W} = 1.80271 , \quad n = 8$$

$$|\sigma_{xx_{sk}}| = 35,511.0 \text{ psi} , \quad |\sigma_{yy_{sk}}| = 2,490.0 \text{ psi}$$

$$\sigma_{xy_{sk}} = 8,370.3 \text{ psi} , \quad |\sigma_{xx_{st}}| = 34,690 \text{ psi}$$

$$\sigma_{yy_r} = 9,229.0 \text{ psi} , \quad S_y = 21,518.0 \text{ psi}$$

$$l_{x_{max}} = 1.600 \text{ in.} , \quad l_{x_{min}} = 1.604 \text{ (MG consideration on } t_{Wx})$$

$$l_{y_{max}} = \text{Any value (from the condition that}$$

$$|\sigma_{xx_{stf_{cr}}} | > |\sigma_{xx_{st}} |)$$

$$l_{y_{min}} = 8.802 \text{ in (MG consideration on } t_y)$$

$$d_{Wx} = .65738 , \quad b_{fx} = .19721$$

$$t_{Wx} = .04986 , \quad d_{fx} = .07368$$

Based on the above data and applying von Mises - Hencky yield criteria we see that yielding of the skin material does not take place. Also there is no yielding of the stiffener materials. Within the bounds as indicated above we adopt

Table F2. Design Chart for Example 3 with TS-RR.

v		z	k_x		k_y	c_x	c_y	\bar{N}_{xy}^*	$\sigma_{xy_{sk}}$	
.33		2221.000	.300		.000	1.079	1.000	.31530E-6	8370.300	
$\bar{\alpha}_x$	$\bar{\alpha}_y$	$\bar{\lambda}_{xx}$	$\bar{\lambda}_{yy}$	\bar{w}	n	$ \sigma_{xx_{sk}} $	$ \sigma_{yy_{sk}} $	$ \sigma_{xx_{st}} $	σ_{yy_r}	s_y
10.0	80.0	.58401	.39940	2.10359	6	33060	3376	31946	7533	20024
10.0	85.0	.55227	.41925	2.09025	6	33787	3567	32609	7582	20363
15.5	55.0	.51541	.31917	1.93658	7	34574	3009	33581	8400	20901
12.5	60.0	.55917	.19933	1.85120	7	33434	2017	32769	9016	20532
12.5	65.0	.55572	.16119	1.80452	7	33471	1692	32913	9354	20630
14.0	50.0	.46519	.27785	1.83384	8	35794	2808	34868	9005	21593
14.0	52.0	.42953	.29099	1.80857	7	36755	2986	35770	9143	22063
14.0	54.0	.41306	.29114	1.79026	7	37211	3024	36213	9256	22297
15.0	45.0	.44693	.30481	1.84360	8	36297	3053	35290	8925	21804
15.0	47.5	.47485	.24044	1.80271	8	35511	2490	34690	9229	21518
15.0	48.0	.42867	.27131	1.78552	8	36762	2831	35827	9300	22102
15.0	49.0	.45336	.23599	1.77360	8	36066	2492	35244	9410	21812
15.0	50.0	.39053	.28073	1.75330	8	37884	2992	36857	9497	22643
16.0	45.0	.41618	.26687	1.76652	8	37103	2822	36172	9422	22287
16.0	47.0	.43646	.22212	1.73906	8	36505	2404	35712	9643	22067
17.5	30.0	.64748	.37980	2.15282	8	31708	3127	30676	7337	19375
17.5	35.0	.43191	.37783	1.90870	8	36758	3612	35566	8518	21921

(Continued)

Table F2. (Continued)

$\bar{\alpha}_x$	$\bar{\alpha}_y$	$l_{x_{\max}}$	$l_{x_{\min}}$	$l_{y_{\max}}$	$l_{y_{\min}}$	d_{wx}	b_{fx}	t_{wx}	t_{fx}	d_{fx}
10.0	80.0	1.660	.869	***	8.924	.43826	.13148	.09534	.09534	.01807
10.0	85.0	1.640	.919	***	9.033	.43826	.13148	.08918	.08918	.02115
12.5	55.0	1.620	1.231	***	7.678	.54782	.16435	.06582	.06582	.04926
12.5	60.0	1.650	1.135	***	13.411	.54782	.16435	.07262	.07262	.04586
12.5	65.0	1.650	1.142	***	17.967	.54782	.16435	.07213	.07213	.04611
14.0	50.0	1.593	1.528	***	8.018	.61356	.18407	.05213	.05213	.06597
14.0	52.0	1.572	1.655	***	7.962	.61356	.18407	.04750	.04750	.06828
14.0	54.0	1.562	1.721	***	8.264	.61356	.18407	.04540	.04540	.06933
15.0	45.0	1.580	1.704	***	6.578	.65738	.19721	.04642	.04642	.07540
15.0	47.5	1.600	1.604	***	8.812	.65738	.19721	.04986	.04986	.07368
15.0	48.0	1.572	1.776	***	7.883	.65738	.19721	.04424	.04424	.07649
15.0	49.0	1.587	1.679	***	9.251	.65738	.19721	.04724	.04724	.07499
15.0	50.0	1.550	1.950	***	7.936	.65738	.19721	.03972	.03972	.07875
16.0	45.0	1.565	1.952	***	7.513	.70121	.21036	.04008	.04008	.08514
16.0	47.0	1.577	1.861	***	9.428	.70121	.21036	.04238	.04238	.08399
17.5	30.0	1.690	1.372	***	3.519	.76695	.23008	.06167	.06167	.08421
17.5	35.0	1.570	2.057	***	4.127	.76695	.23008	.03821	.03821	.09594

*** Shows that $l_{y_{\max}}$ can have any value to satisfy the required constraint.

No. of stringer = 332 , so that $l_x = 1.60865$ in.

No. of rings = 11 , giving $l_y = 9.09091$ in.

Note that the limit $l_{x_{\max}}$ (= 1.600 in.) is based on the selection of the factor k_{l_x} and so is not rigidly adhered to. Clearly, with a different value of k_{l_x} the limit $l_{x_{\max}}$ will be different.

By feeding the above values in the program 'CHECK' of Stage 2, one gets,

$$\bar{N}_{xyP} = 8,717.950 \text{ lb/in} , \quad PB = .04801$$

$$\sigma_{xx}^{st_{Wcr}} = 225,660.0 \text{ psi} , \quad STWB = .15372$$

$$\sigma_{xx}^{st_{fcr}} = 1835,600.0 \text{ psi} , \quad STFB = .01889$$

$$|\sigma_{xy}^{sk_{cr}}| = 8,771.3 \text{ psi} , \quad SB = .95428$$

$$n_p = 30 .$$

Once again it is obvious that l_x and/or l_y can be changed/adjusted within the bounds as found from program 'OPTIMUM' and still all the required constraints will be satisfied. Also the designer has the freedom to move to another point in $\bar{\alpha}_x - \bar{\alpha}_y$ space having the same or very nearly same value of \bar{W} and still can satisfy all the constraints by following the above steps. Once again this confirms that the minimum weight design is not unique.

APPENDIX G

COMPUTER PROGRAMS

The two computer programs, OPTIMUM and CHECK along with the associated subroutines, developed for the combined load cases are given here. OPTIMUM is the main program which first finds a solution for the axial compression with the help of the Subroutines FKXYS, PQSB, NPATR, FUNCTION FF, START and SUMR. Then it seeks an optimum solution for the combined loads by employing the Subroutines START, SUMR, LAGRAN, BBB, IOE, FUNCTION F, EIGEN, ACV and KS. It also calculates the stresses in the skin and stiffeners and shows the bounds on the spacings of the stiffeners in order to satisfy some of the constraints. The final program CHECK finds the panel instability load, $\bar{N}_{xy_{p_{cr}}}$ and checks for the satisfaction of the constraints with the help of the subroutines BBB, IOE, FUNCTION F, EIGEN and ACV. Comment cards have been put in these programs and subroutines to define their purposes. The program has been written in such a way that by removing the comments from desired lines and adding from the undesired ones, all types of stiffeners' geometries that have been considered in this work can be analyzed.

Pertinent program variables and the corresponding mathematical notations are given below. The variables defined in the program have been omitted here.

Program Variable	Mathematical Notation	Program Variable	Mathematical Notation
AL	L		
ALFA,BETA,GAMA	α, β, k	SXXSK	$\sigma_{xx_{sk}}$
CX,CY	C_x, C_y	SYYSK	$\sigma_{yy_{sk}}$
EXB,EYB	\bar{e}_x, \bar{e}_y	SXNB	$\bar{N}_{xy_{p_{cr}}}$
KE1,KE2	m, m'	XL	l_x
N	n	SXSTWC	$\sigma_{xx_{st_{wcr}}}$
RSYY	σ_{yy_r}	SXSTFC	$\sigma_{xx_{st_{fcr}}}$
SKSXY	$\sigma_{xy_{sk}}$	SXYSKC	$\sigma_{xy_{sk_{cr}}}$
STSXX	$\sigma_{xx_{st}}$	VMSXY	σ_s

```

1      C  PROGRAM 'OPTIMUM'
2      C  THIS PROGRAM MINIMIZES THE COMPOSITE WEIGHT FUNCTION (FI
RST    3      C  BY SOLVING FOR AXIAL COMPRESSION ALONE, AND THEN FOR THE
COMBINED 4      C  LOADS) BY THE 'FLEXIBLE POLYHEDRON TYPE OF SIMPLEX' METH
OD      5      C  ALP    =APPLIED LOAD PARAMETER (TORSIONAL)
6      C  ALX    =NONDIMENSIONAL RADIUS OF GYRATION OF STRINGER
7      C  ALY    =NONDIMENSIONAL RADIUS OF GYRATION OF RING
8      C  B      =BETA FOR CIRCUMFERENTIAL WAVES
9      C  CFX    =STRINGER FLANGE-WEB THICKNESS RATIO
10     C  CFY    =RING FLANGE-WEB THICKNESS RATIO
11     C  E      =YOUNG'S MODULUS
12     C  FCX    =STRINGER FLANGE-WIDTH RATIO
13     C  FCY    =RING FLANGE-WIDTH RATIO
14     C  GJ     =(KS-BAR)CR
15     C  H      =SKIN THICKNESS
16     C  SAI    =RATIO OF PRESSURE TO AXIAL LOAD COEFFICIENTS
17     C  STEP   =INITIAL STEP SIZE
18     C  SUM(IN)= COMPOSITE WEIGHT FUNCTION
19     C  U      =POISSON'S RATIO
20     C  WP     =W-BAR (WEIGHT PARAMETER)
21     C  X(I)   =ARRAY OF INITIAL GUESSES
22     C  X(1)   =LAMDA XX BAR

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23      C   X(2)   =LAMDA YY BAR
24      C   X(3)   =LAGRANGE MULTIPLIER FOR AXIAL COMPRESSION
25      C   X(4)   =LAGRANGE MULTIPLIER FOR COMBINED LOAD
26      C   XKB    =AXIAL LOAD COEFFICIENT, (KXX)BAR
27      C   Z      =CURVATURE PARAMETER
28      C   NN     =1 MEANS A SOLUTION FOR AXIAL COMPRESSION IS DESI
RED
29      C   NN     =2 MEANS NO SOLUTION FOR AXIAL COMPRESSION IS DES
IRED
30      DIMENSION X1(10,10),X(10),SUM(10)
31      COMMON/CN1/X1,NX,STEP,K1,SUM,IN
32      COMMON/CN2/ALX,ALY,CX,CY,U,X,Z,XKB,SAI
33      COMMON/CN3/B,GJ,N
34      COMMON/CN4/ALP
35      COMMON/CN7/PRL
36      COMMON/CN8/FCX1,FCY1,FX1,FX2,TY1,TY2
37      COMMON/CN9/EXB,EYB
38      COMMON/CN20/AALP
39      COMMON/CN21/NN
40      COMMON/CN40/XSI
41      COMMON/PPQ/MID
42      COMMON/AAB/MIS(30),KE,MW(250),I2I,VM1(250),VM2(250)
43      COMMON/XX11/XNY
44      DATA/ANB,ANBXY,PRL,ALP,AALP/2700.,418.538,2.67036,3.15
3E-7,
45      12.0339855E-6/
46      C   DATA/ANB,ANBXY,PRL,ALP,AALP/800.,125.,1.031006,0.01862
3E-7,
47      C   11.191941E-8/
48      101 FORMAT(1X,2F6.1,F8.5,F11.3,2F8.5,I3,1X,F8.0,F7.0,F8.0,
F7.0,F8.0,
49      1F7.4,1X,F10.5,I4,2I6,I7)
50      110 FORMAT( )
51      201 FORMAT(1H1,35X,'GENERAL INSTABILITY OPTIMIZATION'//)
52      205 FORMAT(/8X,'NU',5X,'CX',5X,'CY',7X,'Z',6X,'CFX',4X,'CF
Y',4X,
53      1'KS',5X,'KR',7X,'((NXY)BR)SR',7X,'(KXX)BR',9X,'SXYSK')
54      207 FORMAT(6X,F5.3,F6.3,F7.3,3X,F8.1,4F7.4,E15.6,6X,E11.7,
F12.1//)
55      209 FORMAT(4X,'ALX',3X,'ALY',3X,'WB',6X,'(KSB)CR',4X,'X(1)
',4X,
56      1'X(2)',3X,'N',4X,'SXXSK',2X,'SYYSK',4X,'SXXST',3X,'SYY
R',1X,
57      2'VMSXYSK',1X,'MAX LX',3X,'WPSTAR',3X,'II',4X,'MS',4X,'
MF',6X,'M'//)
58      210 FORMAT(4X,'ALX',3X,'ALY',4X,'WB(COMP)',4X,'(KXB)CR',4X
,'XYZ',
59      14X,'YZX',4X,'WB(STAR)',4X,'DIFER',4X,'KI',3X,'ONLY COM
PRESSION'//)
60      211 FORMAT(7X,'DWX',5X,'TWX',5X,'TFX',5X,'BFX',5X,'DFX',4X
,
61      1'MIN LX(.03)',2X,'MIN LY(.03)',5X,'MAX LY',13X,'MIN LX
(G=.02)',
62      24X,'MIN LY(G=.02)'//)
63      216 FORMAT(2X,5F8.5,2F13.5,4X,F9.5,13X,F13.5,4X,F13.5 /)
64      555 FORMAT(1X,2F6.1,F12.5,F11.0,2F8.5,F11.5,1X,E10.5,I3)
65      WRITE(6,201)
66      NX=2
67      STEP=0.1
68      U=0.33

```



```

69      E=0.105E+8
70      READ(5,110)Z
71      C      Z=2221.
72      H=9439.8/(85.*Z)
73      C      SAI IS THE COEFFICIENT FOR LATERAL PRESSURE LOADING (Q
*R/HB)
74      SAI=0.
75      C      SAI=(14.7*85.)/2700.
76      C      XNY IS NYY(=Q*R)
77      XNY=14.7*85.
78      XKB=(12.*2700.*(Z**3)*(85.**3))/((3.1416**2)*0.94398*(
100.**4)
79      1*(10.5*(10.**6)))
80      C      XKB=(12.*800.*(Z**3)*(95.5**3))/((3.1416**2)*0.94398*(
291.**4)
81      C      1*(10.5*(10.**6)))
82      C      H=837.038318/Z
83      SKSXY=ANBXY/H
84      XSI=1./(2.*PRL)
85      CFX=1.
86      CFY=1.
87      FCX=0.3
88      FCY=0.
89      CX=(1.0+2.0*CFX*FCX)/SQRT(1.0+4.0*CFX*FCX)
90      CY=(1.0+2.0*CFY*FCY)/SQRT(1.0+4.0*CFY*FCY)
91      C      FOLLOWING CX AND CY ARE FOR I , Z ,C SECTIONS
92      C      CX=SQRT((1.+2.*CFX*FCX)/(1.+6.*CFX*FCX))
93      C      CY=SQRT((1.+2.*CFY*FCY)/(1.+6.*CFY*FCY))
94      C      FOLLOWING CX AND CY ARE FOR "A-SECTIONS"
95      C      CX=1./SQRT(1.+4.*CFX*FCX)
96      C      CY=1./SQRT(1.+4.*CFY*FCY)
97      C      FOLLOWING ARE FOR HAT SECTIONS
98      C      FCX1=1.
99      C      FCX2=0.2
100     C      FCY1=1.
101     C      FCY2=0.
102     C      TX1=4.+4.*FCX1+3.*FCX2
103     C      TX2=3.*(FCX1**2)+4.*FCX1+8.*FCX2+12.*(FCX2**2)+2.
104     C      TY1=4.+4.*FCY1+3.*FCY2
105     C      TY2=3.*(FCY1**2)+4.*FCY1+8.*FCY2+12.*(FCY2**2)+2.
106     C      XDX1=6.*FCX1**2+8.*FCX1+16.*FCX2+24.*FCX2**2+4.
107     C      YDY1=6.*FCY1**2+8.*FCY1+16.*FCY2+24.*FCY2**2+4.
108     C      CX=2.*(FCX1+1.)/SQRT(XDX1)
109     C      CY=2.*(FCY1+1.)/SQRT(YDY1)
110     WRITE(6,205)
111     WRITE(6,207)U,CX,CY,Z,CFX,CFY,FCX,FCY,ALP,XKB,SKSXY
112     WRITE(6,210)
113     WRITE(6,209)
114     WRITE(6,211)
115     C      ORDER OF DETERMINANT IS DECIDED BELOW FROM CONVERGENCE
116     READ(5,110)KE1,KE2
117     KE=KE2-KE1+1
118     I2=0
119     DO 200 I1=KE1,KE2
120     I2=I2+1
121     200 MIS(I2)=I1
122     C      GUESS STARTING VALUE OF X(1) AND X(2).
123     X(1)=0.2
124     X(2)=0.1
125     X(3)=10.**5

```

```

126      X(4)=1.
127      READ(5,110)ALX,ALY
128      I2I=0
129      C  NP=1 MEANS SOLUTION FOR AXIAL COMPRESSION REQUIRED
130      NN=1
131      EXB=-(((3.1416**2)*(1.0-U**2)**0.5)/(2.0*Z))*(1.0+CX*A
LX)
132      EYB=-(((3.1416**2)*(1.0-U**2)**0.5)/(2.0*Z))*(1.0+CY*A
LY)
133      FG2=1.+(X(1)+X(2))/(1.-U**2)
134      CALL FKXYS(X(1),X(2),ALX,ALY,CX,CY,U,Z,XSI,FKXX)
135      FKKT=FKXX/(Z*Z)
136      FNS=AALP*Z
137      615 FG1=X(3)*(FKKT-FNS)**2
138      FG1=FG1/FG2
139      IF(FG1.GE.1.)GO TO 143
140      X(3)=X(3)*10.
141      GO TO 615
142      143 IF(FG1.LE.50.)GO TO 145
143      X(3)=X(3)/10.
144      GO TO 615
145      XINF=X(3)
146      GO TO 501
147      100 X(1)=1.3*X(1)
148      X(2)=1.5*X(2)
149      IF(X(1).GE.10..AND.X(2).GE.7.)GO TO 777
150      GO TO 666
151      777 WRITE(6,678)ALX,ALY,X(1),X(2)
152      678 FORMAT(1X,'NO SOLUTION IS POSSIBLE FOR ALX=',F8.1,5X,'
ALY=',
153      1F8.1,5X,'X(1)=',F6.3,3X,'X(2)=',F6.3)
154      995 READ(5,110,END=999)ALX,ALY
155      MID=0
156      I2I=0
157      NN=1
158      X(1)=0.2
159      X(2)=0.1
160      EXB=-(((3.1416**2)*(1.0-U**2)**0.5)/(2.0*Z))*(1.0+CX*A
LX)
161      EYB=-(((3.1416**2)*(1.0-U**2)**0.5)/(2.0*Z))*(1.0+CY*A
LY)
162      501 X(3)=XINF
163      559 KI=0
164      C  ALFA, BETA AND GAMA ARE REFLECTION, CONTRACTION AND EX
PANSTON INDICES.
165      ALFA=1.0
166      BETA=0.5
167      GAMA=2.0
168      NX=NX
169      K1=NX+1
170      K2=NX+2
171      K3=NX+3
172      K4=NX+4
173      CALL START
174      DO 3 I=1,K1
175      DO 4 J=1,NX
176      4 X(J)=X1(I,J)
177      IN=I
178      CALL SUMR
179      IF(MID.EQ.1)GO TO 100

```



```

180      3 CONTINUE
181      K5=0
182      63 II=0
183      K5=K5+1
184      28 II=II+1
185      C SELECT LARGEST VALUE OF SUM(I) IN SIMPLEX.
186      60 SUMH=SUM(1)
187      INDEX=1
188      DO 7 I=2,K1
189      IF(SUM(I).LE.SUMH)GO TO 7
190      SUMH=SUM(I)
191      INDEX=I
192      7 CONTINUE
193      C SELECT MINIMUM VALUE OF SUM(I) IN SIMPLEX.
194      SUML=SUM(1)
195      KOUNT=1
196      DO 8 I=2,K1
197      IF(SUML.LE.SUM(I))GO TO 8
198      SUML=SUM(I)
199      KOUNT=I
200      8 CONTINUE
201      C FIND CENTROID OF POINTS WITH I DIFFERENT THAN INDEX.
202      DO 9 J=1,NX
203      SUM2=0.0
204      DO 10 I=1,K1
205      10 SUM2=SUM2+X1(I,J)
206      X1(K2,J)=1.0/XNX*(SUM2-X1(INDEX,J))
207      C FIND REFLECTION OF HIGH POINT THROUGH CENTROID.
208      X1(K3,J)=(1.0+ALFA)*X1(K2,J)-ALFA*X1(INDEX,J)
209      IF(X1(K3,J).LT.0.0)X1(K3,J)=0.0
210      9 X(J)=X1(K3,J)
211      IN=K3
212      CALL SUMR
213      IF(MID.EQ.1)GO TO 100
214      IF(SUM(K3).LT.SUML)GO TO 11
215      C SELECT SECOND LARGEST VALUE IN SIMPLEX.
216      IF(INDEX.EQ.1)GO TO 38
217      SUMS=SUM(1)
218      GO TO 39
219      38 SUMS=SUM(2)
220      39 DO 12 I=1,K1
221      IF((INDEX-I).EQ.0)GO TO 12
222      IF(SUM(I).LE.SUMS)GO TO 12
223      SUMS=SUM(I)
224      12 CONTINUE
225      IF(SUM(K3).GT.SUMS)GO TO 13
226      GO TO 14
227      C FORM EXPANSION OF NEW MINIMUM IF REFLECTION HAS PRODUC
ED ONE MINI.
228      11 DO 15 J=1,NX
229      X1(K4,J)=(1.0-GAMA)*X1(K2,J)+GAMA*X1(K3,J)
230      IF(X1(K4,J).LT.0.0)X1(K4,J)=0.0
231      15 X(J)=X1(K4,J)
232      IN=K4
233      CALL SUMR
234      IF(MID.EQ.1)GO TO 100
235      IF(SUM(K4).LT.SUML)GO TO 16
236      GO TO 14
237      13 IF(SUM(K3).GT.SUMH)GO TO 17
238      DO 18 J=1,NX

```



```

239      18 X1(INDEX,J)=X1(K3,J)
240      C      FORM CONTRACTION IF REFLECTION PRODUCED AN ABSOLUTE MA
XIMUM.
241      17 DO 19 J=1,NX
242          X1(K4,J)=BETA*X1(INDEX,J)+(1.0-BETA)*X1(K2,J)
243          IF(X1(K4,J).LT.0.0)X1(K4,J)=0.0
244      19 X(J)=X1(K4,J)
245          IN=K4
246          CALL SUMR
247          IF(MID.EQ.1)GO TO 100
248          IF(SUMH.GT.SUM(K4))GO TO 16
249      C      REDUCE SIMPLEX BY HALF WHEN REFLECTION PRODUCED AN ABS
OLITE MAXI.
250          DO 20 J=1,NX
251          DO 20 I=1,K1
252      20 X1(I,J)=0.5*(X1(I,J)+X1(KOUNT,J))
253          DO 29 I=1,K1
254          DO 30 J=1,NX
255      30 X(J)=X1(I,J)
256          IN=I
257          CALL SUMR
258          IF(MID.EQ.1)GO TO 100
259      29 CONTINUE
260          GO TO 26
261      16 DO 21 J=1,NX
262          X1(INDEX,J)=X1(K4,J)
263      21 X(J)=X1(INDEX,J)
264          IN=INDEX
265          CALL SUMR
266          IF(MID.EQ.1)GO TO 100
267          GO TO 26
268      14 DO 22 J=1,NX
269          X1(INDEX,J)=X1(K3,J)
270      22 X(J)=X1(INDEX,J)
271          IN=INDEX
272          CALL SUMR
273          IF(MID.EQ.1)GO TO 100
274      26 DO 23 J=1,NX
275      23 X(J)=X1(K2,J)
276          IN=K2
277          CALL SUMR
278          IF(MID.EQ.1)GO TO 100
279      C      TO TERMINATE THE SEARCH, DIFER MUST BE LESS THAN EPSIL
ON.
280          DIFER=0.
281          DO 25 I=1,K1
282      25 DIFER=DIFER+(SUM(I)-SUM(K2))*2
283          DIFER=SQRT(1./(XNX+1.)*DIFER)
284          IF(DIFER.GE.0.01.AND.II.LT.30)GO TO 28
285          IF(NN.EQ.2)GO TO 888
286          CALL FKXYS(X1(KOUNT,1),X1(KOUNT,2),ALX,ALY,CX,CY,U,Z,X
SI,FKXX)
287          IF(K5.GE.9)GO TO 862
288          WWP=1.+(X1(KOUNT,1)+X1(KOUNT,2))/(1.-U**2)
289          EG1=SUML-WWP
290          EG2=ABS(EG1/X(3))
291          A5=ABS(EG1/WWP)
292          A6=ABS(SQRT(EG2)/FNS)
293          IF(A5.LT.0.001.AND.A6.LT.0.001)GO TO 862
294          A8=10.

```



```

295      IF(K5.GT.1.AND.A6.LT.0.001.AND.ABS((WWP-A7)/WWP).LT.0.
001)A8=0.1
296      A7=WWP
297      IF(A6.GT.0.01)A8=100.
298      KI=KI+II
299      DO 293 I=1,3
300      DO 294 J=1,2
301      294 X(J)=X1(I,J)
302      PPWD=1.+(X(1)+X(2))/(1.-U**2)
303      293 SUM(I)=(SUM(I)-PPWD)*A8+PPWD
304      X(3)=X(3)*A8
305      TKKF=FKXX/(Z*Z)
306      GO TO 63
307      862 KI=KI+II
308      XYZ=X1(KOUNT,1)
309      YZX=X1(KOUNT,2)
310      WBB=1.+(XYZ+YZX)/(1.-U**2)
311      WRITE(6,555)ALX,ALY,WBB,FKXX,XYZ,YZX,SUML,DIFER,KI
312      C OPTIMIZATION FOR AXIAL COMPRESSION COMPLETED HERE
313      X(1)=1.2*XYZ
314      X(2)=1.2*YZX
315      666 IF(X(4).LT.2.)GO TO 333
316      C NN=2 MEANS SOLUTION FOR COMBINEDD LOAD TO PROCEED
317      331 NN=2
318      X(4)=YIMF
319      GO TO 559
320      333 CALL LAGRAN(SNF,YIMF)
321      GO TO 331
322      888 IF(K5.GE.9)GO TO 762
323      WP=1.0+(X1(KOUNT,1)+X1(KOUNT,2))/(1.0-U**2)
324      FGG1=SUML-WP
325      FGG2=ABS(FGG1/X(4))
326      A1=ABS(FGG1/WP)
327      A2=ABS(SORT(FGG2)/SNF)
328      IF(A1.LT.0.001.AND.A2.LT.0.001)GO TO 762
329      A4=10.
330      IF(K5.GT.1.AND.A2.LT.0.001.AND.ABS((WP-A3)/WP).LT.0.00
1)A4=0.1
331      A3=WP
332      IF(A2.GT.0.01)A4=100.
333      KI=KI+II
334      DO 193 I=1,3
335      DO 194 J=1,2
336      194 X(J)=X1(I,J)
337      PWD=1.+(X(1)+X(2))/(1.-U**2)
338      193 SUM(I)=(SUM(I)-PWD)*A4+PWD
339      X(4)=X(4)*A4
340      FKKKT=GJ/(Z**2)
341      GO TO 63
342      762 KI=KI+II
343      BET=B
344      XA=X1(KOUNT,1)
345      XB=X1(KOUNT,2)
346      C OPTIMIZATION FOR COMBINED LOAD COMPLETED HERE
347      SXXSK= ((ANB/H)*(((SAI/2.-1.)*(1.+XB-U**2)+U*SAI*XA)/
(1.+XA)
348      1*(1.+XB)-U**2)))
349      SYYSK= ((ANB/H)*((SAI*(1.+XA)-SAI*(U**2)+U*XB*(SAI/2.-
1.))/
350      1((1.+XA)*(1.+XB)-U**2)))

```



```

351          STSXX= ((ANB*(1.-U**2)/H)*(((SAI/2.-1.)*(1.+XB)-U*SAI)
/((1.+XA)
352          1*(1.+XB)-U**2)))
353          RSYX=(ANB*(1.-U**2)/H)*(((SAI*(1.+XA)-U*(SAI/2.-1.))/((
1.+XA)
354          1*(1.+XB)-U**2)))
355          VMSXY=SQRT((SXXSK**2+SYYSK**2-SXXSK*SYYSK)/3.+SKSXY*SK
SXY)
356          C      FOR A ROUGH ESTIMATE OF MAXIMUM LX VALUE SYYSK AND SXY
SK ARE NEGLECTED
357          C      CHANGE CONSTANT IN THE FOLLOWING EQUATION AS DESIRED
358          XLMAX=0.968*H*SQRT(((3.1416**2)*E)/(3.*(1.-U**2)*ABS(S
XXSK)))
359          C      FOLLOWING EXPRESSIONS ARE FOR TS-RR AS WELL AS AS-RR
360          DWX=(H*ALX*(1.+CFX*FCX))/SQRT(1.+4.*CFX*FCX)
361          TWX=(XLMAX*XA*H)/(DWX*(1.-U**2)*(1.+CFX*FCX))
362          TFX=CFX*TWX
363          BFX=FCX*DWX
364          C      FOLLOWING DFX IS FOR T-STRINGER
365          DFX=(BFX-TWX)/2.
366          C      FOLLOWING DFX IS FOR A-STRINGER
367          C      DFX=DWX-TFX
368          TEST=(12.*(1.-U**2)*DFX*DFX*ABS(STSXX))/((3.1416**2)*E
*TFX*TFX)
369          1-0.407
370          C      FOLLOWING EXPRESSIONS ARE FOR CS,ZS,IS-RR
371          C      DWX=H*ALX*SQRT((1.+2.*CFX*FCX)/(1.+6.*CFX*FCX))
372          C      TWX=(XLMAX*XA*H)/(DWX*(1.-U**2)*(1.+2.*CFX*FCX))
373          C      TFX=CFX*TWX
374          C      BFX=FCX*DWX
375          C      DFX=BFX/2.
376          C      TEST=(12.*(1.-U**2)*DFX*DFX*STSXX)/((3.1416**2)*E*TFX*
TFX)-0.407
377          IF(TEST.LT.0.)GO TO 214
378          YLMAX=(0.95*DFX)/SQRT(TEST)
379          C      FOLLOWING MODIFICATION IN YLMAX IS FOR HR ONLY
380          C      YLMAX=YLMAX+FCY1*((H*ALY*(FCY1+2.+2.*FCY2))/SQRT(YDY1)
)
381          GO TO 215
382          C      FOLLOWING EXPRESSIONS ARE FOR HS-RR
383          C      DWX=(H*ALX*(FCX1+2.+2.*FCX2))/SQRT(XDX1)
384          C      TWX=(XLMAX*XA*H)/(DWX*(1.-U**2)*(FCX1+2.+2.*FCX2))
385          C      TFX=TWX
386          C      BFX=FCX1*DWX
387          C      DFX=BFX
388          214 YLMAX=999999.
389          C      THE FOLLOWING IS FOR MINIMUM GAUGE=0.05 WITH T-STRINGER
390          215 XLMIN=(0.05*ALX*(1.-U**2)*(1.+CFX*FCX)**2)/(XA*SQRT(1.
+
391          14.*CFX*FCX))
392          C      FOLLOWING XLMIN IS FOR C,Z,I-STRINGERS
393          C      215 XLMIN=(0.05*ALX*(1.-U**2)*(1.+2.*CFX*FCX)*SQRT(1.+2.*C
FX*FCX))
394          C      1/(XA*SQRT(1.+6.*CFX*FCX))
395          C      FOLLOWING XLMIN AND YLMIN ARE FOR HS AND HR
396          C      XLMIN=(0.05*ALX*(1.-U**2)*(FCX1+2.+2.*FCX2)**2)/(XA*SQ
RT(XDX1))
397          C      YLMIN=(0.05*ALY*(1.-U**2)*(FCY1+2.+2.*FCY2)**2)/(XB*SQ
RT(YDY1))
398          C      FOLLOWING YLMIN IS FOR RR AND MG=0.05

```



```

399      YLMIN=(0.05*ALY*(1.-U**2))/XB
400      C FOLLOWING XXLMIN AND YYLMIN ARE FOR MG=.04 WITH TS-RR
401      XXLMIN=(0.04*ALX*(1.-U**2)*(1.+CFX*FCX)**2)/(XA*
402      1SQRT(1.+4.*CFX*FCX))
403      YYLMIN=(0.04*ALY*(1.-U**2))/XB
404      C FOLLOWING YLMIN IS FOR T-RING AND A-RING
405      C YLMIN=(0.05*ALY*(1.-U**2)*(1.+CFY*FCY)**2)/(XB*SQRT(1.
+
406      14.*CFY*FCY))
407      C FOLLOWING YLMIN IS FOR C,Z,I-RINGS
408      C YLMIN=(0.05*ALY*(1.-U**2)*(1.+2.*CFY*FCY)*SQRT(1.+2.*C
FY*FCY))
409      C 1/(XB*SQRT(1.+6.*CFY*FCY))
410      C FOLLOWING EXPRESSIONS ARE FOR HS ONLY
411      C SXST2C=(3.1416*3.1416*E*TWX*TWX)/(3.*(1.-U**2)*FCX1*FC
X1*DWX*DWX)
412      C SXST1C=SXST2C*(FCX1*FCX1)
413      C SXST1C=(3.1416*3.1416*E*TWX*TWX)/(3.*(1.-U**2)*(DWX-TW
X)**2)
414      C FOLLOWING MODIFICATION IN XLMAX IS FOR HS ONLY
415      C XLMAX=XLMAX+BFX
416      C WRITE(6,101)ALX,ALY,WP,GJ,XA,XB,N,SXXSK,SYYSK,STSXX,RS
YY,
417      1VMSXY,XLMAX,SUML,KI,KE1,KE2,MW(I2I)
418      C WRITE(6,216)DWX,TWX,TFX,BFX,DFX,XLMIN,YLMIN,YLMAX,XXLM
IN,YYLMIN
419      C THE FOLLOWING CHECKS EIGENVECTORS FOR CONVERGENCE
420      DO 753 I1=1,I2I
421      IF((MW(I1)).LE.KE1+1.AND.KE1.NE.1).OR.(MW(I1)).GE.KE2-1)
.OR.(ABS
422      1(VM1(I1)).GT.0.3.AND.KE1.NE.1).OR.(ABS(VM2(I1)).GT.0.3
))GO TO 57
423      GO TO 58
424      57 WRITE(6,754)I1,MW(I1),VM1(I1),VM2(I1)
425      GO TO 995
426      58 CONTINUE
427      754 FORMAT(//,2X,'CHECK EIGENVECTOR I=',I6,2X,'MW=',I6,
428      12X,'VM1(I1)=',E12.4,2X,'VM2(I1)=',E12.4)
429      753 CONTINUE
430      GO TO 995
431      999 CONTINUE
432      STOP
433      END

```



```

1      SUBROUTINE FKXYS(XLAMD,YLAMD,ALFX,ALFY,
2      1CX,CY,XNI,Z,XSI,FKXX)
3      C THIS FINDS BUCKLING LOAD COEFFICIENT IN
4      C AXIAL COMPRESSION
5      C
6      COMMON/CN9/EXB,EYB
7      COMMON/XXXX/B(3),P(8),S(3),Q,ZR,XSS,XSP
8      M=1
9      N=1
10     MD=2
11     ND=2
12     RHOX=ALFX**2*XLAMD
13     RHOY=ALFY**2*YLAMD
14     CALL PQSB(XLAMD,YLAMD,RHOX,RHOY,XNI,Z
15     1,XSI,EXB,EYB)
16     CALL NPATR(N,M,ND,MD,FKXX)
17     RETURN
18     END

```

```

1      SUBROUTINE PQSB(XLAMD,YLAMD,RHOX,RHOY
2      1,XNI,Z,XSI,EX,EY)
3      C THIS IS FOR FINDING SOLUTION FOR
4      C AXIAL COMPRESSION
5      COMMON/XXXX/B(3),P(8),S(3),Q,ZR,XSS,XSP
6      XSS=XSI**2
7      XSP=3.1416**2/(32.*XSI)
8      XSS2=XSS**2
9      B(1)=1.+XLAMD
10     B(2)=2./(1.-XNI)*(B(1)*(1.+YLAMD)-XNI)*XSS
11     B(3)=(1.+YLAMD)*XSS2
12     P(1)=1.+RHOX
13     P(2)=2.*XSS
14     P(3)=(1.+RHOY)*XSS2
15     ZR=12.*Z**2/(3.1416**4*(1.-XNI**2))
16     P(4)=EX**2*XLAMD
17     P(5)=2.*P(4)*(1.-XNI+YLAMD)/(1.-XNI)*XSS
18     P(6)=(P(4)*(1.+YLAMD)+2.*EX*EY*XLAMD*YLAMD
19     1*(1.+XNI))/(1.-XNI)
20     1+EY**2*YLAMD*B(1))*XSS2
21     P(7)=2./(1.-XNI)*EY**2*YLAMD*(B(1)-XNI)
22     1*XSS**3
23     P(8)=EY**2*YLAMD*XSS2**2
24     Q=(B(1)*(1.+YLAMD)-XNI**2)
25     S(1)=XNI*EX*XLAMD
26     S(2)=-(EX*XLAMD*(1.+YLAMD)+EY*YLAMD
27     1*(1.+XLAMD))*XSS
28     S(3)=XNI*EY*YLAMD*XSS2
29     RETURN
30     END

```



```

1      SUBROUTINE NPATR(N,M,ND,MD,FX)
2      C THIS IS FOR FINDING A SOLUTION FOR
3      C AXIAL COMPRESSION
4      DIMENSION NZ(2),MZ(2)
5      K=0
6      NZ(1)=N
7      MZ(1)=M
8      NR=ND
9      MR=MD
10     NA1=0
11     NA2=0
12     A1=NZ(1)
13     A2=MZ(1)
14     FX=FF(A1,A2)
15     NZ(2)=NZ(1)+NA1+NR
16     K=K+1
17     IF(K.LT.70)GO TO 450
18     WRITE(6,460)K,FX,NZ(1),MZ(1)
19     460 FORMAT(//,2X,'K ITER=',I5,3X,'FX=',
20     1E12.4,3X,'N=',I5,3X,'M=',I5)
21     STOP
22     450 IF(NZ(2).LE.0)NZ(2)=1
23     MZZ=MZ(1)+NA2
24     IF(MZZ.LE.0)MZZ=1
25     A1=NZ(2)
26     A2=MZZ
27     F1=FF(A1,A2)
28     IF(F1.LT.FX)GO TO 10
29     NZ(2)=NZ(1)+NA1-NR
30     IF(NZ(2).LE.0)NZ(2)=1
31     A1=NZ(2)
32     F1=FF(A1,A2)
33     IF(F1.LT.FX)GO TO 10
34     NR=NR-1
35     IF(ABS(NR).GT.0)GO TO 15
36     F1=FX
37     NZ(2)=NZ(1)+NA1
38     IF(NZ(2).LE.0)NZ(2)=1
39     10 FX=F1
40     25 MZ(2)=MZ(1)+NA2+MR
41     IF(MZ(2).LE.0)MZ(2)=1
42     A1=NZ(2)
43     A2=MZ(2)
44     F1=FF(A1,A2)
45     IF(F1.LT.FX)GO TO 20
46     MZ(2)=MZ(1)+NA2-MR
47     IF(MZ(2).LE.0)MZ(2)=1
48     A2=MZ(2)
49     F1=FF(A1,A2)
50     IF(F1.LT.FX)GO TO 20
51     MR=MR-1
52     IF(ABS(MR).GT.0)GO TO 25
53     F1=FX
54     MZ(2)=MZ(1)+NA2
55     IF(MZ(2).LE.0)MZ(2)=1
56     IF(NA1.EQ.0.AND.NA2.EQ.0.AND.NR.EQ.0)
57     1GO TO 1000
58     20 FX=F1
59     IF(NR.EQ.0.AND.MR.EQ.0)GO TO 250

```



```

60      NA1=NZ(2)-NZ(1)
61      NA2=MZ(2)-MZ(1)
62      GO TO 150
63      250 NZ(2)=NZ(2)-NA1
64      MZ(2)=MZ(2)-NA2
65      NA1=0
66      NA2=0
67      150 NZ(1)=NZ(2)
68      MZ(1)=MZ(2)
69      NR=NR+1
70      MR=MR+1
71      GO TO 15
72      1000 N=NZ(2)
73      M=MZ(2)
74      RETURN
75      END

```

```

1      FUNCTION FF(RN, RM)
2      C THIS IS FOR FINDING SOLUTION FOR AXIAL
3      C COMPRESSION
4      COMMON/XXXX/B(3), P(8), S(3), Q, ZR, XSS, XSP
5      COMMON/XX11/XNY
6      RN2=RN*RN
7      RN4=RN2*RN2
8      RN6=RN4*RN2
9      RN8=RN4*RN4
10     RM2=RM*RM
11     RM4=RM2*RM2
12     RM6=RM4*RM2
13     B1=B(1)*RM4+B(2)*RM2*RN2+B(3)*RN4
14     ZRB=ZR/B1
15     P1=P(1)*RM2+P(2)*RN2+P(3)*RN4/RM2
16     P2=ZRB*(P(4)*RM6+P(5)*RM4*RN2+P(6)*RM2
17     1*RN4+P(7)*RN6+P(8)*RN8/RM2
18     2+2.*S(1)*RM4+2.*S(2)*RM2*RN2+2.*S(3)
19     3*RN4+Q*RM2)
20     FF=P1+P2
21     XX=RN2/RM2*XSS
22     FF=FF-XX*XNY
23     RETURN
24     END

```



```

1      SUBROUTINE START
2      C      SET UP THE INITIAL SIMPLEX FROM ONE
3      C      STARTING POINT.
4      DIMENSION X1(10,10),X(10),SUM(10),D(10,10)
5      COMMON/CN1/X1,NX,STEP,K1,SUM,IN
6      COMMON/CN2/ALX,ALY,CX,CY,U,X,Z,XKB,SAI
7      VN=NX
8      STEP1=STEP/(VN*SQRT(2.0))*(SQRT(VN+1.0)
9      1+VN-1.0)
10     STEP2=STEP/(VN*SQRT(2.0))*(SQRT(VN+1.0)
11     1-1.0)
12     DO 1 J=1,NX
13     1 D(1,J)=0.0
14     DO 2 I=2,K1
15     DO 2 J=1,NX
16     D(I,J)=STEP2
17     L=I-1
18     D(I,L)=STEP1
19     2 CONTINUE
20     DO 3 I=1,K1
21     DO 3 J=1,NX
22     3 X1(I,J)=X(J)+D(I,J)
23     RETURN
24     END

```

```

1      SUBROUTINE SUMR
2      C      SUMR IS THE COMPOSITE WEIGHT EXPRESSION.
3      DIMENSION X1(10,10),X(10),SUM(10)
4      COMMON/CN1/X1,NX,STEP,K1,SUM,IN
5      COMMON/CN2/ALX,ALY,CX,CY,U,X,Z,XKB,SAI
6      COMMON/CN3/B,GJ,N
7      COMMON/CN4/ALP
8      COMMON/CN7/PRL
9      COMMON/CN9/EXB,EYB
10     COMMON/CN20/AALP
11     COMMON/CN21/NN
12     COMMON/CN40/XSI
13     COMMON/CN50/FKXX
14     COMMON/PPQ/MTD
15     COMMON/AAB/MIS(30),KE,MW(250),I2I,
16     1VM1(250),VM2(250)
17     DO 10 J=1,NX
18     10 IF(X(J).LT.0.0)X(J)=0.0
19     IF(NN.EQ.2)GO TO 15
20     CALL FKXYS(X(1),X(2),ALX,ALY,CX,CY,U
21     1,Z,XSI,FKXX)
22     SUM(IN)=1.+(X(1)+X(2))/(1.-U**2)+X(3)
23     1*(FKXX/(Z**2)-AALP*Z)**2
24     GO TO 20
25     15 CALL KS
26     IF(MID.EQ.1)GO TO 20
27     SUM(IN)=1.0+(X(1)+X(2))/(1.0-U**2)+X(4)
28     1*(GJ/(Z**2)-Z*ALP)**2
29     20 RETURN
30     END

```



```

1      SUBROUTINE LAGRAN(SNF,YIMF)
2      C      THIS FINDS LAGRANGE MULTIPLIER FOR FIRST
3      C      DATA POINT OF COMBINED LOAD
4      DIMENSION X(10)
5      COMMON/CN2/ALX,ALY,CX,CY,U,X,Z,XKB,SAI
6      COMMON/CN3/B,GJ,N
7      COMMON/CN4/ALP
8      COMMON/PPQ/MID
9      COMMON/AAB/MIS(30),KE,MW(250),I2I,
10     1VM1(250),VM2(250)
11     X(4)=10.**5
12     GF2=1.+(X(1)+X(2))/(1.-U**2)
13     CALL KS
14     IF(MID.EQ.1)GO TO 20
15     TKKF=GJ/(Z*Z)
16     SNF=ALP*Z
17     15 GF1=X(4)*(TKKF-SNF)**2
18     GF1=GF1/GF2
19     IF(GF1.GE.1.)GO TO 43
20     X(4)=X(4)*10.
21     GO TO 15
22     43 IF(GF1.LE.50.)GO TO 45
23     X(4)=X(4)/10.
24     GO TO 15
25     45 YIMF=X(4)
26     20 RETURN
27     END

```

```

1      FUNCTION IOE(LN,LM)
2      C      THIS HELPS SETTING TERMS IN THE DETERMINANT
3      C      BUT FOR THE DIAGONAL TERMS
4      L1=(LN+LM)/2
5      L1=2*L1
6      IOE=1
7      IF(L1.EQ.LN+LM)IOE=2
8      RETURN
9      END

```



```

1      SUBROUTINE BBB(BB, KK)
2      C THIS IS FOR FINDING ALL THE TERMS IN THE
3      C DETERMINANT
4      C BUT FOR THE DIAGONAL TERMS
5      DIMENSION BB(KK, KK)
6      COMMON/AAB/MIS(30), KE, MW(250), I2I, VM1(250)
7      1, VM2(250)
8      BB(KE, KE)=0.
9      K1=KE-1
10     DO 2 I=1, K1
11     BB(I, I)=0.
12     L=I+1
13     DO 2 J=L, KE
14     IF (IOE(MIS(I), MIS(J)).EQ.2) GO TO 50
15     A1=MIS(I)*MIS(J)
16     A2=MIS(J)**2-MIS(I)**2
17     II=MIS(I)/2
18     Q=MIS(I)/2.
19     IA=2*II
20     IB=2*Q
21     IF (IA.EQ.IB) GO TO 20
22     GO TO 10
23     20 BB(I, J)=-A1/A2
24     GO TO 2
25     10 BB(I, J)=A1/A2
26     GO TO 2
27     50 BB(I, J)=0.
28     2 CONTINUE
29     DO 4 I=2, KE
30     L=I-1
31     DO 4 J=1, L
32     4 BB(I, J)=BB(J, I)
33     RETURN
34     END

1      FUNCTION F(B, M)
2      C THIS IS FOR EVALUATING THE DIAGONAL TERMS WITH KS-BAR
3      DIMENSION X(10)
4      COMMON/CN2/ALX, ALY, CX, CY, U, X, Z, XKB, SAI
5      COMMON/CN5/AL1, AL2, AL3, AL4, AL5, AL6, AL7, AL11, AL12, AL13
6      COMMON/CN6/XLB, YLB, RBX, RBY
7      COMMON/CN7/PRL
8      COMMON/CN9/EXB, EYB
9      AL8=(1.0+RBX)*(M**4)+2.0*AL7*(M**2)*(B**2)+(1.0+RBY)*
B**4)
10     AL9=XLB*(EXB**2)*(M**8)+AL2*(M**6)*(B**2)+AL3*(M**4)*
B**4)+AL4*(M
11     1**2)*(B**6)+(EYB**2)*YLB*(B**8)-2.0*U*EXB*XLB*(M**6)+2
.0*AL5*(M**4)

```



```

12      2)*(B**2)-2.0*U*EYB*YLB*(M**2)*(B**4)+AL6*(M**4)
13      AL10=(1.0+XLB)*(M**4)+AL1*(M**2)*(B**2)+(1.0+YLB)*(B**
4)
14      C      AL14=((EXB*XLB*(M**6))/(1.+XLB)+AL11*(M**4)*(B**2)+AL1
2      1*(M**2)*(B**4)+EYB*YLB*(B**6)/(1.+XLB)+(M**4)*U/(1.+XL
B)-AL13
16      C      2*(M**2)*(B**2)+(1.+YLB)*(B**4)/(1.+XLB))/((PRL**2)*(M*
*4+AL1*
17      C      3(M**2)*(B**2)/(1.+XLB)+(1.+YLB)/(1.+XLB)))+(M**2)*0.5+
B**2
18      F=((3.1416)/(8.0*B))*(AL8+(12.0*(Z**2)*AL9)/((3.1416**
4)*(1.0-
19      1U**2)*AL10)-XKB*(M**2-SAI))
20      C      FOLLOWING CONTINUATION CARD IS FOR PRESSURE CASE ONLY
21      C      1U**2)*AL10)-XKB*(M**2-SAI*AL14))
22      RETURN
23      END

```

```

1      SUBROUTINE EIGEN(VA,B,K,V0,V1,V2,KK,ERR
2      1,FKSS)
3      C      THIS IS FOR FINDING THE LOWEST EIGENVALUE
4      C      (I.E. BUCKLING LOAD COEFFICIENT IN TORSION)
5      DIMENSION B(KK,KK),VA(KK),V0(KK),V1(KK)
6      1,V2(KK)
7      COMMON/PPQ/MTD
8      MTD=0
9      FKSS=0.
10     C      INITIAL VECTOR ASSIGNED HERE
11     DO 1 I=1,K
12     1 V0(I)=1.
13     IMODE=1
14     155 CALL ACV(K,K,V1,B,V0,KK)
15     DO 2 I=1,K
16     2 V1(I)=-V1(I)/VA(I)
17     CALL ACV(K,K,V2,B,V1,KK)
18     DO 3 I=1,K
19     3 V2(I)=-V2(I)/VA(I)
20     RQN=0.
21     RQD=0.
22     DO 14 I=1,K
23     RQN=RQN+V0(I)*V2(I)
24     14 RQD=RQD+V2(I)*V2(I)
25     C      THE EIGEN VALUE IS OBTAINED FROM THE
26     C      RAYLEIGH QUOTIENT
27     RQSS=FKSS
28     R11=RQN/RQD
29     IF(IMODE.GT.1) GO TO 510
30     FKSS=SQRT(ABS(R11))
31     GO TO 520
32     510 IF(R11.GE.0.) GO TO 530
33     FKSS=-SQRT(ABS(R11))
34     RETURN
35     530 FKSS=SQRT(R11)
36     520 CONTINUE

```



```

37      IF (ABS((ROSS-FKSS)/FKSS)-ERR)160,160,50
38      50 IMODE=IMODE+1
39      IF (IMODE-15)150,170,170
40      150 FMAX=0.
41          DO 17 I=1,K
42          IF (ABS(FMAX)-ABS(V2(I)))16,17,17
43      16 FMAX=V2(I)
44      17 CONTINUE
45          DO 18 I=1,K
46      18 V0(I)=V2(I)/FMAX
47          GO TO 155
48      170 MID=1
49          GO TO 10
50      C 170 WRITE(6,300)IMODE
51      C 300 FORMAT(//,2X,'NUMBER OF ITERATIONS
52      C   GREATER THAN',I5,2X,'AND THE
53      C   1SEQUENCE OF EIGENVALUE PROBLEM',/5X,
54      C   'DOES NOT CONVERGE WITHIN ACCUR
55      C   2ACY REQUIRED BY ERR'//)
56      160 CONTINUE
57          CALL ACV(K,K,V1,B,V2,KK)
58          DO 29 I=1,K
59      29 V1(I)=-V1(I)/VA(I)
60          DO 31 I=1,K
61      31 V0(I)=V2(I)+FKSS*V1(I)
62          V1(I)=V2(I)-FKSS*V1(I)
63          FMAX=0.
64          FMAX1=0.
65          DO 33 I=1,K
66      33 IF (ABS(FMAX)-ABS(V0(I)))41,44,44
67      41 FMAX=V0(I)
68      44 IF (ABS(FMAX1)-ABS(V1(I)))42,33,33
69      42 FMAX1=V1(I)
70      33 CONTINUE
71          DO 39 I=1,K
72      39 V1(I)=V1(I)/FMAX1
73          V0(I)=V0(I)/FMAX
74      10 RETURN
75      END
  
```

```

1      SUBROUTINE ACV(N,M,C,A,R,NM)
2      C THIS HELPS FINDING THE LOWEST EIGENVALUE
3      C (IN EIGEN)
4      DIMENSION C(NM),A(NM,NM),R(NM)
5      C C(N)=A(N,M)*R(M)
6      DO 30 I=1,N
7      C(I)=0.
8      DO 30 J=1,M
9      30 C(I)=C(I)+A(I,J)*R(J)
10     RETURN
11     END
  
```



```

1      SUBROUTINE KS
2      C THIS IS FOR FINDING BUCKLING LOAD COEFFICIENT IN TORSION
3      DIMENSION X(10)
4      DIMENSION BB(50,50),VA(50)
5      DIMENSION V0(50),V1(50),V2(50)
6      COMMON/CN2/ALX,ALY,CX,CY,U,X,Z,XKB,SAI
7      COMMON/CN3/B,GJ,N
8      COMMON/CN5/AL1,AL2,AL3,AL4,AL5,AL6,AL7,AL11,AL12,AL13
9      COMMON/CN6/XLB,YLB,RBX,RBY
10     COMMON/CN7/PRL
11     COMMON/CN8/FCX1,FCY1,FX1,FX2,TY1,TY2
12     COMMON/CN9/EXB,EYB
13     COMMON/PPQ/MID
14     COMMON/AAB/MIS(30),KE,MW(250),I2I,VM1(250),VM2(250)
15     XLB=X(1)
16     YLB=X(2)
17     KK=50
18     ERR=0.01
19     CALL RBB(BB,KK)
20     RBX=(ALX**2)*XLB
21     RBY=(ALY**2)*YLB
22     AL1=(2.0/(1.0-U))*((1.0+XLB)*(1.0+YLB)-U)
23     AL2=(2.0*(EXB**2)*XLB)*(1.0+YLB-U)/(1.0-U)
24     AL3=(EXB**2)*XLB+(EYB**2)*YLB+XLB*YLB*(EXB**2+EYB**2+2
    .0*EXB*EYB*(
25     11.0+U)/(1.0-U))
26     AL4=(2.0*(EYB**2)*YLB*(1.0+XLB-U))/(1.0-U)
27     AL5=EXB*XLB+EYB*YLB+(EXB+EYB)*XLB*YLB
28     AL6=(1.0+XLB)*(1.0+YLB)-(U**2)
29     AL7=1.0
30     C AL7=1.+XLB*(ALX**2)*(12./(1.+U))*((FCX1**2)/(TX1*TX2))
31     C 1+YLB*(ALY**2)*(12./(1.+U))*((FCY1**2)/(TY1*TY2))
32     C AL11=((EXB*XLB)/(1.+XLB))*(1.-U+(2.*YLB)/(1.-U))
33     C AL12=((EYB*YLB)/(1.+YLB))*(1.+(2.*XLB)/(1.-U))
34     C AL13=((1.+U)*(1.-U+YLB)+2.*XLB*(1.+YLB))/((1.-U)*(1.+X
    LB))
35     C1=2.
36     B1=C1/PRL
37     DO 15 I=1,KE
38     15 VA(I)=F(B1,MIS(I))
39     CALL EIGEN(VA,BB,KE,V0,V1,V2,KK,ERR,GJ1)
40     IF(MID.E0.1)GO TO 200
41     DO 80 N=4,100,2
42     C2=N
43     B2=C2/PRL
44     DO 20 J=1,KE
45     20 VA(J)=F(B2,MIS(J))
46     CALL EIGEN(VA,BB,KE,V0,V1,V2,KK,ERR,GJ2)
47     IF(MID.E0.1)GO TO 200
48     IF(GJ1-GJ2)85,85,75
49     75 C1=C2
50     B1=B2
51     GJ1=GJ2
52     80 CONTINUE
53     85 C3=C1+1.
54     B3=C3/PRL
55     DO 25 K=1,KE
56     25 VA(K)=F(B3,MIS(K))
57     CALL EIGEN(VA,BB,KE,V0,V1,V2,KK,ERR,GJ3)

```



```

58      IF (MID.EQ.1) GO TO 200
59      IF (GJ1-GJ3) 90,95,95
60      90 GJ=GJ1
61      B=B1
62      N=C1
63      GO TO 110
64      95 GJ=GJ3
65      B=B3
66      N=C3
67      110 DO 1 IJ=1,KE
68      IF (ABS(V0(IJ)).GE.0.9999) GO TO 2
69      1 CONTINUE
70      2 I2I=I2I+1
71      MW(I2I)=MIS(IJ)
72      VM1(I2I)=V0(1)
73      VM2(I2I)=V0(KE)
74      200 RETURN
75      END

```

OVER BOND
SECTION FIBER
USA

```

1      C PROGRAM "CHECK"
2      C THIS PROGRAM FINDS PANEL INSTABILITY LOAD BY
3      C GOLDEN SECTION METHOD AND CHECKS FOR THE
4      C INEQUALITY CONSTRAINTS.
5      C USING FIBONACCI FRACTIONS. (FIBONACCI FRACTION(F1)=0.3
82).
6      DIMENSION X1(100),X2(100),X3(100),Y1(100),Y2(100),DEL(
100)
7      DIMENSION X(10)
8      DIMENSION BB(50,50),VA(50)
9      DIMENSION V0(50),V1(50),V2(50)
10     COMMON/CN2/ALX,ALY,CX,CY,U,X,Z,XKB,SAI
11     COMMON/CN5/AL1,AL2,AL3,AL4,AL5,AL6,AL7,AL11,AL12,AL13
12     COMMON/CN6/XLB,YLB,RBX,RBY
13     COMMON/CN7/PRL
14     COMMON/CN9/EXB,EYB
15     COMMON/PPQ/MID
16     COMMON/AAB/MIS(30),KE,MW(250),I2I,VM1(250),VM2(250)
17     DATA X1(1),X2(1),X3(1),F1,EPS/0.1,2.0,4.0,0.381966011,
0.01/
18     201 FORMAT(1H1,30X,'PANEL INSTABILITY LOAD AND CRITICAL ST
RESSES
19     1ARE AS FOLLOWS'//)
20     400 FORMAT(5X,'Z',7X,'L',7X,'BETA',5X,'N',7X,'(NXY)BR',5X,
'PB',5X,
21     1'LX',5X,'(SXX)STWCR',3X,'(SXX)STFCR',5X,'(SXY)SKCR',7X
,'SB',
22     29X,'M',1X,'MX EG VECT.'//)
23     405 FORMAT(2X,F6.0,2X,F8.5,F7.3,I6,E14.6,F9.7,F7.3,2E13.5,
E14.5,
24     1F11.5,I8,1X,F11.7//)
25     410 FORMAT(
26     WRITE(6,201)
27     U=0.33
28     E=0.105E+8
29     R=85.0

```



```

30      ANBXY=418.538
31      C      R=95.5
32      C      ANBXY=125.
33      KK=50
34      ERR=0.01
35      WRITE(6,400)
36      C      SAI IS THE COEFFICIENT FOR LATERAL PRESSURE LOADING (Q
      *R/N)
37      SAI=0.
38      C      SAI=(14.7*85.)/2700.
39      KE1=1
40      KE2=5
41      KE=KE2-KE1+1
42      I2=0
43      DO 200 I1=KE1,KE2
44      I2=I2+1
45      200 MIS(I2)=I1
46      CALL BBB(BB,KK)
47      C      XL(=LX) IS THE ADOPTED STRINGER SPACING
48      408 READ(5,410,END=999)ZZ,ALX,XLB,AL,XL,SXXSK
49      I2I=0
50      H=9439.8/(ZZ*R)
51      C      H=79937.17038/(ZZ*R)
52      SKSXY=ANBXY/H
53      PRL=(3.1416*R)/AL
54      Z=((AL**2)*SQRT(1.0-U**2))/(R*H)
55      D=((10.5*(10.0**6))*H**3)/(12.0*(1.0-U**2))
56      C      XKB IS THE AXIAL LOAD COEFFICIENT((NB*L**2)/(PAI**2)*D)
57      C
58      XKB=(2700.*(AL**2))/((3.1416**2)*D)
59      C      XKB=(800.*(AL**2))/((3.1416**2)*D)
60      C      CFX,CFY,FCX,FCY SHOULD BE DEFINED AND CX AND CY SHOULD
      BE FOUND
61      C      BELOW FOR THE PARTICULAR TYPE STRINGERS AND RINGS USED

62      C      FOLLOWING CX & CY ARE FOR RECT. & T SECTIONS
63      CFX=1.
64      CFY=1.
65      FCX=0.3
66      FCY=0.
67      CX=(1.0+2.0*CFX*FCX)/SQRT(1.0+4.0*CFX*FCX)
68      CY=(1.0+2.0*CFY*FCY)/SQRT(1.0+4.0*CFY*FCY)
69      C      FOLLOWING CX AND CY ARE FOR I , Z SECTIONS
70      C      CX=SQRT((1.+2.*CFX*FCX)/(1.+6.*CFX*FCX))
71      C      CY=SQRT((1.+2.*CFY*FCY)/(1.+6.*CFY*FCY))
72      C      FOLLOWING CX AND CY ARE FOR "ANGLE SECTIONS"
73      C      CX=1./SQRT(1.+4.*CFX*FCX)
74      C      CY=1./SQRT(1.+4.*CFY*FCY)
75      C      FOLLOWING ARE FOR HAT SECTIONS
76      C      FCX1=1.
77      C      FCX2=0.
78      C      FCY1=1.
79      C      FCY2=0.
80      C      TX1=4.+4.*FCX1+3.*FCX2
81      C      TX2=3.*(FCX1**2)+4.*FCX1+8.*FCX2+12.*(FCX2**2)+2.
82      C      TY1=4.+4.*FCY1+3.*FCY2
83      C      TY2=3.*(FCY1**2)+4.*FCY1+8.*FCY2+12.*(FCY2**2)+2.
84      C      XDX1=6.*FCX1**2+8.*FCX1+16.*FCX2+24.*FCX2**2+4.
85      C      YDY1=6.*FCY1**2+8.*FCY1+16.*FCY2+24.*FCY2**2+4.
86      C      CX=2.*(FCX1+1.)/SQRT(XDX1)
87      C      CY=2.*(FCY1+1.)/SQRT(YDY1)

```



```

88      C      THE FOLLOWING VALUES ARE FOR 'T & A-STRINGERS'
89      DWX=(H*ALX*(1.+CFX*FCX))/SQRT(1.+4.*CFX*FCX)
90      TWX=(XL*XLB*H)/(DWX*(1.-U**2)*(1.+CFX*FCX))
91      TFX=CFX*TWX
92      BFX=FCX*DWX
93      C      FOLLOWING DFX IS FOR T-STRINGER
94      DFX=(BFX-TWX)/2.
95      C      FOLLOWING DFX IS FOR A-STRINGER
96      C      DFX=DWX-TFX
97      C      FOLLOWING ARE FOR "C,Z,I-STRINGERS"
98      C      DWX=H*ALX*SQRT((1.+2.*CFX*FCX)/(1.+6.*CFX*FCX))
99      C      TWX=(XL*XLB*H)/(DWX*(1.-U**2)*(1.+2.*CFX*FCX))
100     C      TFX=CFX*TWX
101     C      BFX=FCX*DWX
102     C      DFX=BFX/2.
103     C      FOLLOWING ARE FOR HS
104     C      DWX=(H*ALX*(FCX1+2.+2.*FCX2))/SQRT(XDX1)
105     C      TWX=(XL*XLB*H)/(DWX*(1.-U**2)*(FCX1+2.+2.*FCX2))
106     C      TFX=TWX
107     C      BFX=FCX1*DWX
108     C      DFX=BFX
109     C
110     YLB=0.0
111     ALY=0.0
112     C      POSITIVE EXB MEANS OUTSIDE STRINGERS AND NEGATIVE MEAN
S INSIDE.
113     EXB=-(((3.1416**2)*(1.0-U**2)**0.5)/(2.0*Z))*(1.0+CX*AL
LB)
114     EYB=0.0
115     RBX=(ALX**2)*XLB
116     RBY=0.0
117     AL1=(2.0/(1.0-U))*((1.0+XLB)*(1.0+YLB)-U)
118     AL2=(2.0*(EXB**2)*XLB)*(1.0+YLB-U)/(1.0-U)
119     AL3=(EXB**2)*XLB+(EYB**2)*YLB+XLB*YLB*(EXB**2+EYB**2+2
.0*EXB*EYB*(
120     11.0+U)/(1.0-U))
121     AL4=(2.0*(EYB**2)*YLB*(1.0+XLB-U))/(1.0-U)
122     AL5=EXB*XLB+EYB*YLB+(EXB+EYB)*XLB*YLB
123     AL6=(1.0+XLB)*(1.0+YLB)-(U**2)
124     AL7=1.0
125     C      AL7=1.+XLB*(ALX**2)*(12./(1.+U))*((FCX1**2)/(TX1*TX2))
126     C      1+YLB*(ALY**2)*(12./(1.+U))*((FCY1**2)/(TY1*TY2))
127     C      AL11=((EXB*XLB)/(1.+XLB))*(1.-U+(2.*YLB)/(1.-U))
128     C      AL12=((EYB*YLB)/(1.+YLB))*(1.+(2.*XLB)/(1.-U))
129     C      AL13=((1.+U)*(1.-U+YLB)+2.*XLB*(1.+YLB))/(1.-U)*(1.+X
LB)
130     C      GOLDEN SECTION METHOD STARTS HERE
131     K=1
132     LL=0
133     DO 15 I=1,KE
134     15 VA(I)=F(X2(K),MIS(I))
135     CALL EIGEN(VA,BR,KE,V0,V1,V2,KK,ERR,EF1)
136     IF(MID.EQ.1)GO TO 555
137     DO 20 J=1,KE
138     20 VA(J)=F(X3(K),MIS(J))
139     CALL EIGEN(VA,BB,KE,V0,V1,V2,KK,ERR,EF2)
140     IF(MID.EQ.1)GO TO 555
141     71 IF(EF1-EF2)70,70,80
142     80 X3(K)=X3(K)+0.2*X3(K)
143     IF(X3(K).LT.15.)GO TO 71

```



```

144      LL=LL+1
145      IF (LL.LT.10)GO TO 71
146      X1(1)=0.00001
147      X2(1)=0.8
148      X3(1)=1.0
149      IF (LL.LT.11)GO TO 71
150      70 DEL(K)=X3(K)-X1(K)
151      72 Y1(K)=X1(K)+F1*DEL(K)
152      Y2(K)=X3(K)-F1*DEL(K)
153      C MINIMIZATION WITH RESPECT TO BETA
154      DO 25 I=1,KE
155      25 VA(I)=F(Y1(K),MIS(I))
156      CALL EIGEN(VA,BB,KE,V0,V1,V2,KK,ERR,FE1)
157      IF (MID.EQ.1)GO TO 555
158      DO 30 J=1,KE
159      30 VA(J)=F(Y2(K),MIS(J))
160      CALL EIGEN(VA,BB,KE,V0,V1,V2,KK,ERR,FE2)
161      IF (MID.EQ.1)GO TO 555
162      IF (FE1-FE2)90,91,92
163      555 WRITE (6,560)ZZ,AL,ALX,XLB
164      560 FORMAT(1X,'BUCKLING TAKES PLACE WITH ONLY AXIAL LOAD A
T',5X,'Z=',
165      1F7.0,3X,'L=',F5.0,3X,'ALX=',F5.0,3X,'X(1)=',F8.5)
166      GO TO 408
167      90 DEL(K+1)=Y2(K)-X1(K)
168      X1(K+1)=X1(K)
169      X3(K+1)=Y2(K)
170      K=K+1
171      IF (ABS((X3(K)-X1(K))/X3(K)).LT.EPS)GO TO 95
172      GO TO 72
173      91 DEL(K+1)=Y2(K)-X1(K)
174      X1(K+1)=Y1(K)
175      X3(K+1)=X3(K)
176      K=K+1
177      IF (ABS((X3(K)-X1(K))/X3(K)).LT.EPS)GO TO 95
178      GO TO 72
179      92 DEL(K+1)=X3(K)-Y1(K)
180      X1(K+1)=Y1(K)
181      X3(K+1)=X3(K)
182      K=K+1
183      IF (ABS((X3(K)-X1(K))/X3(K)).LT.EPS)GO TO 95
184      GO TO 72
185      C MEAN VALUE OF BETA AT (KS)CR.
186      95 B=(X1(K)+X3(K))/2.0
187      C WITH KNOWN PRL WE FIND THE INTEGER VALUE OF N FOR (KS)
CR.
188      C B1 IS THE VALUE OF N(NUMBER OF CIRCUMFERENTIAL WAVES).
189      B1=PRL*B
190      C
191      IF (B1-2.0)97,97,98
192      C IF N>2 THEN FIND (KS)CR AT INTEGER VALUE OF N
193      98 I1=B1
194      I2=I1+1
195      C RB1 AND RB2 ARE THE INTEGER VALUES OF N ADJACENT TO (B
ETA)CR.
196      RB1=I1
197      RB2=I2
198      C NEXT B1 AND B2 ARE THE VALUES OF BETA AT THE TWO ADJAC
ENT INTEGER N.
199      B1=RB1/PRL
200      B2=RB2/PRL

```



```

201      DO 35 I=1,KE
202      35 VA(I)=F(B1,MIS(I))
203      CALL EIGEN(VA,BB,KE,V0,V1,V2,KK,ERR,X11)
204      IF (MID.EQ.1)GO TO 555
205      DO 40 J=1,KE
206      40 VA(J)=F(B2,MIS(J))
207      CALL EIGEN(VA,BB,KE,V0,V1,V2,KK,ERR,X12)
208      IF (MID.EQ.1)GO TO 555
209      IF (X11-X12)100,100,105
210      C   IF N<2 THEN FIND ((KS(BAR))CR. AT.N=2
211      97 B1=2.0
212      B=B1/PRL
213      DO 45 I=1,KE
214      45 VA(I)=F(B,MIS(I))
215      CALL EIGEN(VA,BB,KE,V0,V1,V2,KK,ERR,GJ)
216      IF (MID.EQ.1)GO TO 555
217      XYNB=((3.1416**2)*D*GJ)/(AL**2)
218      N=2
219      GO TO 110
220      100 GJ=X11
221      XYNB=((3.1416**2)*D*GJ)/(AL**2)
222      N=RB1
223      B=B1
224      GO TO 110
225      105 GJ=X12
226      XYNB=((3.1416**2)*D*GJ)/(AL**2)
227      N=RB2
228      B=B2
229      C   FOLLOWING VALUES ARE FOR "T-STRINGERS"
230      110 SXSTWC=(3.1416*3.1416*E*TWX*TWX)/(3.*(1.-U**2)*DWX*DWX
)
231      C   FOLLOWING SXSTWC IS FOR "C,Z,I-STRINGERS"
232      C 110 SXSTWC=(3.1416*3.1416*E*TWX*TWX)/(3.*(1.-U**2)*(DWX-2.
*TFX)**2)
233      SXSTFC=((3.1416*3.1416*E)/(12.*(1.-U**2)))*((TFX/DFX)*
*2)*
234      1((DFX/AL)**2+0.407)
235      C   FOLLOWING SXSTWC AND SXSTFC ARE FOR A-STRINGERS
236      C 110 SXSTWC=((3.1416*3.1416*E)/(12.*(1.-U**2)))*((TWX/DFX)*
*2)*
237      C   1((DFX/AL)**2+0.407)
238      C   SXSTFC=0.99999E+38
239      C   FOLLOWING SXSTFC AND SXSTWC ARE FOR H-STRINGERS
240      C 110 SXSTFC=(3.1416*3.1416*E*TWX*TWX)/(3.*(1.-U**2)*FCX1*FC
X1*DWX*DWX)
241      C   SXSTWC=SXSTFC*FCX1*FCX1.
242      C   SXSTWC=(3.1416*3.1416*E*TWX*TWX)/(3.*(1.-U**2)*
1(DWX-TWX)*(DWX-TWX))
243      C   1(DWX-TWX)*(DWX-TWX))
244      ETA=SXXSK/SKSXY
245      C   FOLLOWING ARE FOR HAT-STIFFENERS ONLY
246      C   XL=XL-BFX
247      C   AL=AL-BFY
248      AALF=AL/XL
249      IF (AALF.LT.1.)GO TO 210
250      COEF=4./3.+1./(AALF**2)
251      SXYSKC=((3.1416**2)*E)/(6.*(1.-U**2))*((H/XL)**2)*ET
A*
252      1(COEF**2)*(SQRT(1.+(2./(ETA*COEF))**2)-1.)
253      GO TO 215

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254      210 COEF=(4.*(AALF**2)+5.34)/((AALF*AALF+1.）**2)
255      SXYSKC=((3.1416**2)*E)/(24.*(1.-U**2)))*((H/XL)**2)*E
TA*
256      1(COEF**2)*(SQRT(1.+(2./(ETA*COEF))**2)-1.)*((AALF+1./A
ALF)**2)
257      215 SB=SKSXY/SXYSKC
258      C FOLLOWING MODIFICATIONS IN 'SXYSKC' AND 'SB' ARE MADE FO
R
259      C COMBINED LOAD CASE 'NXX+NXY+Q' - VALUES BROUGHT FROM OT
HER PROGRAM
260      C READ(7,500)SXYSKC,SB
261      500 FORMAT(1X,F10.0,F12.5)
262      PB=ANBXY/XYNB
263      DO 1 IJ=1,KE
264      IF (ABS(V0(IJ)).GE.0.9999)GO TO 2
265      1 CONTINUE
266      2 I2I=I2I+1
267      MW(I2I)=MIS(IJ)
268      WRITE(6,405)ZZ,AL,B,N,XYNB,PB,XL,SXSTWC,SXSTFC,SXYSKC,
SB,MW(I2I),
269      1V0(IJ)
270      GO TO 408
271      999 CONTINUE
272      STOP
273      END

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PERMANIZED
 PLOVER BOND

25% COTTON FIBER

U.S.A.

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